Search Space Analysis for the Combined Mathematical Model (Linear and Nonlinear) of the Water Distribution Network Design Problem

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Abstract. This paper presents an experimental study of the solutions space generated by the mathematical model of the Water Distribution Network Design Problem by using Two-Looped network benchmarks to find the feasible solutions space. It shows how the performance of a typical Evolutionary Algorithm (EA) can be improved by considering the importance of working with a feasible population and carrying out repetitive mutations and crossovers to generate new feasible offspring with better fitness. The replacement of parents represents the mortality index of a population at each generation of EA. Aiming to compensate the mortality index, EA is forced to maintain a constant population size by increasing the number of descendants with the crossover operator. The experimental results show both the feasible solutions space and the results of the algorithm when using feasible solutions and varying population size.

1. Introduction

In life, there are problems with several solutions and one must be chosen. This is the case for combinatory optimization problems [1].

An optimization problem has some important characteristics; it has an objective function to be optimized, a search space, and a subset of the search space. The feasible solutions space for combinatory optimization problems is a discrete set, or it can be reduced to a discrete set.

The Water Distribution Network Design problem (WDND) is an optimization problem. It consists of finding the most efficient way to supply water to consumers, within given constraints. For example pressure requirements must be reached to offer users an adequate service when satisfying their water requirements. The WDND Problem has been widely studied by many researchers. The first attempts to solve the problem were based on Lineal Programming techniques. Alperovits and Shamir [7] proposed a linear programming gradient method which has been adapted and improved by Quindry [8], Goulter et al. [9], Fujiwara et al. [10], and Kessler et al. [11], among others. It is noteworthy that the previously cited works present similarities in their mathematical formulation, decision variables, and methods used to solve the problem. The mathematical formulation was based on lineal programming models, the decision variables was based on continuous variables, and the solution method for the problem was primarily based on lineal programming methods. The design of the network tended to be a branched layout. In the last decade, the WDND problem has gradually been modified. It has been formulated as a non linear programming problem and pipe diameters have been stated as discrete decision variables. The solution method for the problem has generally been based on heuristic methods like Evolutionary Algorithms (EAs), Simulated Annealing and others. The design of the network has been a looped layout, and the network technique to supply water to consumers has been gravity. Even though the problem has been referred to as the WDND problem for three decades, there are some important differences between the first two decades and last decade. These differences alter the problem slightly, and do not allow for direct comparison. They include mathematical formulation, decision variables, topology (branched or looped), solution method, and technique to feed the network (pumping or gravity).

According to the computational complexity theory, WDND is verified as an NP-Complete problem by mapping it to the well-known Job Shop Scheduling Problem [3]. It is classified in the set of NP-Hard problems [4], and has been widely studied over 30 years by many researchers due to its practical application. In order to solve this problem, several approaches have been applied. When trying to solve the WDND problem, global optimization [5, 6], linear programming [7, 8], non-linear programming [9, 10, 11, 12, and 13] and many other heuristics have been applied [14, 15, 16, 17]. When attempts are made to solve this problem for real instances, it is extremely complex to find the optimum solution. Even for small benchmarks of NP-Complete problems, finding the global optimum solution by using an exact method would take years [2]. A good alternative is the use of heuristic methods. One of the most promising and commonly used methods is the well-known EAs. These methods are stochastic search procedures, based on evolution and natural selection [21, 21]. They suggest a satisfactory success rate for identifying good solutions. They have successfully handled NP-Complete problems [18, 19] for different fields, including the WDND problem [20, 23]. An EA consists of 5 main components: 1) Solution Representation, 2) Initial Population, 3) Evaluation Function, 4) Genetic Operators and 5) Parametric values for population size, crossover and mutation probabilities, and number of generations. Recently, many works have focused on developing EAs. When working with an EA to solve the WDND problem, some questions related to the components of the EA arise: What percentage of the feasible solutions is included in the complete search space? How many solutions should be generated to find a feasible solution? What must the size of the feasible initial population be in order to have a representative sample of the search space? What method is used to create an initial population?

In order to find the global optimal solution, it is important to know the size of the feasible solutions space. The goal is to know if the size and characteristics of the initial population help the Evolutionary Algorithm to converge earlier to a better solution. These questions are addressed in this article.

In this paper, an EA, called EA-WDND, is presented. EA-WDND differs from traditional EAs in four important aspects: 1) Initial population creation. It is a subset of a feasible solutions space, all the individuals of the population can be selected to generate offspring. 2) The population size of offspring generated is bigger than the population size of parents. 3) For each generation, the population is created by the best offspring; parents are combined to produce offspring and then they die. Unfeasible individuals cannot survive. 4) EA-WDND algorithm solves two models: the constraints satisfaction model by using Epanet Solver, and the optimization model by evaluating the objective function.

The principal contribution of this work is the experimental study of the search space of the WDND Problem. It helps determine how many solutions should be generated, and the time needed to obtain different sizes of feasible populations. The study shows the difficulty of finding a feasible solution in the complete solutions space. An experimental study of an evolutionary algorithm, EA-WDND, presented here, shows convergence by using different sizes of initial feasible populations.

This paper is organized as follows: Section 2 explains the combined Mathematical Model for the WDND Problem. Section 3 presents a description of the Evolutionary Algorithm. Section 4 defines the Solutions Space for the WDND Problem. Section 5 describes the experimental results of the solutions space. Section 6 presents the conclusions and future investigations to provide continuity to this work.

2. Water Distribution Network Design Problem (WDND)

The optimization of the looped water distribution networks is an important and complex problem with applications in urban, industrial and irrigation water supply. It consists of minimizing the network investment cost with pipe diameters as decision variables, while link layout, connectivity, and demands are imposed as constraints [24]. The solution to the problem is the least cost optimum configuration, which is a sequence of the necessary pipe diameters to convey water from sources to all the network water users, satisfying their requirements.

Recently, the model that represents the WDND problem has been stated as a non-programming lineal model, and hydraulic restrictions have been managed as implicit restrictions [13]. In this work, the mathematical model represents looped networks and has been divided into two models to classify design restrictions, independent of operation restrictions: 1) the model of lineal programming includes network design restrictions which can be stated mathematically in terms of the cost of a pipeline and unit length for each pipeline (Table 1 and 2). The constraints satisfaction model includes network operation restrictions.

Equation (1) is the objective function. It consists of minimizing the total cost, $T_{C,}$ of the water distribution network configuration, where n is the number of pipes in the network. T_C is based on the sum of the costs of each pipe of length L_{ijdk}. Cost C_{ijdk} is taken from a commercial diameters list and it depends directly of the diameter of pipe used. The cost of a pipeline is assumed to be linearly proportional to its length. The objective function is subject to constraints set. Constraints in (2) indicate that one or more pipes L_{ijdk} in the network can have the same diameter d_k. At the same time, it indicates when a diameter included in the set D of commercial diameters, is not being used for a pipe in the network. D is the set of commercial diameters available for the water network design, $D = \{d_1, d_2, ..., d_n\}$. Constraints in (3) indicate that each node i in the network can be connected to pipes of length L_{ijdk} with the same or different commercial diameter sizes. Constraints in (4) indicate that for each pipe, of length $\mathrm{L}_{\mathrm{ijd}_\mathrm{k}}$, a single pipe diameter of the list of commercially available diameters must be used. Restrictions in (5) define values that can be assigned to the set of variables X. For example, when considering reference equation (1), if a pipe connected from node i to node j uses a diameter d_k then $X_{ijdk} = 1$, otherwise $X_{ijdk} = 0$.

Table 1. Model of Lineal Programming

$$\min T_c = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{d_k=d_1}^{d_n} C_{ijd_k} L_{ijd_k} X_{ijd_k} \qquad (1)$$

Subject to:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} X_{ijd_k} \geq 0 \qquad \forall (d_k = d_1, ..., d_n) \qquad (2)$$

$$\sum_{i=1}^{n}\sum_{d_k=d_1}^{d_n} X_{ijd_k} \geq 1 \qquad \forall (j = 1, ..., n) \qquad (3)$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{d_k=d_1}^{d_n} X_{ijd_k} = 1 \qquad (4)$$

$$X_{ijd_k} \in \{0,1\} \qquad \forall (i, j, k) \qquad (5)$$

The constraints satisfaction model (Table 2) includes network operation restrictions. They refer to the necessary restrictions to operate a looped water network properly. Constraint (6) represents the physical law of mass conservation on each of *n* nodes of the network, where Q_{in} are the pipe flows into the loop, Q_{out} are the pipe flows away from the loop, and Q_e is positive if it is an external demand and negative if it is a supply. The flow entering a node must be equal to the flow leaving the node. Constraint (7) refers to the law of conservation of energy in a mesh m; in this case m is a loop in the network. It indicates that the sum of the frictional energy losses along pipe lengths belonging to the hydraulic mesh should be zero if there are not power pumps in m. Constraint (8) refers to the minimum and maximum pressure requirements to satisfy the users' water requirements while guaranteeing appropriate network operation. Pressure requirements are verified at each demand node *i* of the network. Finally, constraint (9) is related to the limitation of flow velocity V in pipes. The minimum velocity requirement is defined to avoid reducing the diameter of pipes because of sediments. The maximum velocity requirement helps to reach required pressures.

Table 2. Constraints Satisfaction Model

$$\sum_i Q_{in} - \sum_i Q_{out} = Q_e \qquad \forall\ i \in n \qquad (6)$$

$$\sum_m h_f - \sum_m E_p = 0 \qquad \forall m \qquad (7)$$

$$H_{\min} \leq H_i \leq H_{\max} \qquad \forall\ i \in n \qquad (8)$$

$$V_{\min} \leq V_{L_{ij}} \leq V_{\max} \qquad \forall L_{ij} \qquad (9)$$

3. Evolutionary Algorithm

Evolutionary Algorithms (EAs) are adaptive methods which attempt to imitate the biological and genetic processes and can successfully be applied to optimization problems. The main fields of application of EAs include problems such as Water Distribution Networks, with high complexity, non-linear behavior, and a high number of decision variables [25]. EAs are stochastic numerical search procedures inspired by biological evolution allowing the individuals with better fitness to survive and propagate their genes to successive generations. EAs deal with a population of individuals, which experience constant changes by means of genetic operators like reproduction, crossover, and mutation. EAs are gaining popularity due to their capabilities in handling several real world problems involving complexity, noisy environments, imprecision, uncertainty, and vagueness [26].

In this work, for the WDND problem, the individuals of a population are represented by a set of parameters (commercial diameters and lengths of pipes) that describe a solution. Each solution is codified into a chromosome structure to represent the analogy with the characters strings. They are evaluated with respect to the objective function in (1) and ranked according to their fitness. The best individuals for the problem are those individuals with least-cost. Generally, the best individuals are more likely to be candidate solutions to reproduce, having offspring that compose the next generation.

Figure 1 shows the solution methodology used to solve the WDND problem by using an evolutionary algorithm. The proposed algorithm in this work, called EA-WDND, works in Linux platforms. It uses the well-known Epanet Solver [27] version 2.0 [28] to verify hydraulic constraints, Table 2.

The solutions space (SS), also known in the literature as search space, includes all possible solutions to the problem. The size of the SS depends directly on the input instance analyzed. Hence, for two-looped network instances, the search space would include 1,875,000,000 possible solutions. SS includes feasible and unfeasible solutions. Feasible solutions are those solutions that obey restrictions of the lineal programming model and restrictions of the hydraulic model at the same time (section 2). Unfeasible solutions are those solutions that do not obey all constraints included in both models.

An instance of a WDND problem is defined by the function $f: SS \rightarrow R$, where SS is the finite set of solutions that defines the problem instance, R is the set of real values that defines each solution in SS, and f is the objective function. In a problem instance, it is necessary to find the solution $s \in SS$ for which $f(s) \leq f(y), \forall y \in SS$, where s is feasible. The set *R* includes decision variables which are discrete values; specifically it refers to pipe diameters. In Figure 1, $FU = \{ s | s \in SS, FU \subseteq SS \}$ is a subset taken from *SS*. *FU* can contain feasible and unfeasible solutions because restrictions of the hydraulic model are not considered at this point. The set of feasible solutions space is represented by $FS = \{s | s \text{ is feasible}, FS \subseteq FU, FS \neq FU\}$. The set *FS* considers both the constraints of the lineal and the hydraulic model. *FS* is created by taking solutions of *FU* and verifying them to determine whether they obey hydraulic constraints. The verification is done using the EPANET Solver. Therefore, *FS* can only include feasible solutions.

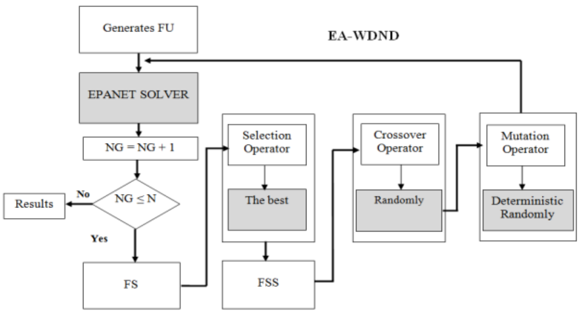


Fig. 1. Solution Methodology for the WDND Problem

It is known that the initial feasible population, which is not necessarily the best one, allows good individuals in next generations of the genetic algorithm to be obtained. When generating the initial population, a question arises regarding its optimal size. The selection operator used is "*the best*" (elitist) [29]. It consists of taking the best individuals of the population *FS*. According to their fitness, the operator "the best" selects an average of the best individual values from a population. Then individuals are combined producing offspring that will compose the next generation, called the Feasible Solutions Subset *FSS* (see Figure 1), $FSS = \{s | s \in FS, FSS \subseteq FS, FSS \neq FS\}$. *FSS* has the same definition for the feasible solutions space *FS*. The difference between *FS* and *FSS* is that *FS,* in the first generation of the algorithm, contains feasible individuals randomly generated. *FSS* contains offspring of individuals included into the *FS* set. For the next generations, *FS* is created by replacing its individuals with offspring that result from applying crossover and mutation operators. It is important to mention that the number of crossover or mutations is directly related to the population size. For each individual of the population, a crossover or mutation is applied. Consequently, the number of feasible offspring individuals included in *FS* is slightly larger than the size of *FSS*. Some descendants are eliminated because they are not feasible when Epanet evaluates them. The feasible individuals are kept in a temporary list and they are ranked according to their fitness. The fittest offspring replace parents at each generation to constitute a new feasible population set, *FSS.* The *FSS* set is used at each generation to carry out crossover or mutation on its individuals.

The crossover operator is a function $Cr(s_1, s_2) \xrightarrow{\sigma} (s_1', s_2')$; it consists of exchanging σ chromosome information of the two parents (s_1, s_2) to produce an offspring pair (s_1', s_2') that inherits characteristics of the parents. $FS = \{(s_1', s_2') | (s_1', s_2') \in FS, (s_1', s_2') \in SS\}$. The crossover σ refers to the combination of two feasible solutions, s_1 and s_2, to generate two new individuals, s'_1 and s'_2. These new individuals are then verified in Epanet to determine whether they are feasible solutions. The crossing strategy implemented in this work is called one point cross-over [24]. It generates two offspring, the s'_1 and s'_2 chromosomes. To determine whether the offspring chromosomes are better than their parents, their fitness has to be computed with the objective function (see Eq. 1). In the EA-WDND algorithm, the parents are removed and replaced by the best offspring to keep a stable population size. The result is a new generation, usually with better fitness.

The mutation operator [24] involves randomly replacing a targeted gene. The mutation operator is a function, $M(s) \xrightarrow{\alpha} (s')$. The mutation α, implemented in the mutation module, consists of randomly replacing the targeted gene using a random number K ϵ [1, n], where n is the total number of genes in the chromosome. Each gene represents a pipe diameter. It is replaced with a random integer K [d_1, d_n], where n is the total number of commercial diameters. For each individual mutated, an offspring chromosome is generated and a deterministic mutation α is carried out. The mutation operator α involves randomly selecting a gene to be mutated, using a random number K ϵ [1, n], where n is the total number of genes in the chromosome. It is replaced with the gene of greatest diameter that is located in the next position of the array (i+1). Another variation consists of randomly selecting a gene to be mutated, using a random number K ϵ [1, n], where n is the total number of genes in the chromosome. The randomly selected gene is replaced with the gene of smallest diameter located in the (i-1) position of the array.

4. Solutions Space

In order to have a representative sample of the population space, an experimental study was conducted. It consisted of generating solutions for the WDND problem. The objective was to determine the percentage of feasible solutions for this problem. The experimental study was carried out based on the Two-Looped network benchmark [7].

[13] The Two-Looped network has seven nodes and eight pipes arranged in two loops. The network is fed by the gravity technique. It has a fixed head reservoir of 210 m. The pipes are 1000 m in length. The minimum pressure limitation is 30 m above ground level for each node. There are 14 commercial diameters which can be selected. The nodal head and demands, the cost per meter for each size of pipe, and other data are widely reported in many previous works [7, 30, 31, 8, and 32].

In the literature, information on how to define the size of initial population for the WDND problem was not found. Some researches use various population sizes, Table 3.

Table 3. Population Size

Date	Researchers	Population Size
1997	Savic et al.	50
1999	Montesinos et al.	300
2003	Matias et al.	100-1000
2006	Reca et al.	500

5. Experimental Results

The experimental study for the WDND problem involved the generation of different population sizes to know the number of feasible individuals (verified in Epanet) that can be obtained for each sample. Additionally, for each sample, the time required to obtain feasible populations was measured. To generate a feasible population, the algorithm was executed 30 times for each defined sample population. Table 4, shows the results obtained from the executions of the algorithm. After 30 executions were carried out, the average for a sample of 15,000 individuals was 144 feasible individuals generated in 12 seconds.

Table 4. Population Size

Sample	Feasible Individuals	Time (sec.)
15,000	144	12
30,000	293	24
60,000	589	63
120,000	1188	98
240,000	2374	186
480,000	4752	383
960,000	9509	720

1,920,000	19355	1500

According to the obtained results, it can be noted that the feasible solutions space is 0.01% of the complete solutions space for the benchmark Two-Looped network, Fig. 2a. Based on the experimental results, it can be deduced that the time needed to generate the complete solutions space (1,475,800,000) should be approximately 521 hours, Fig. 2b. The required time increases according to the input instance, so the algorithm could spend years generating all possible solutions for larger instances.

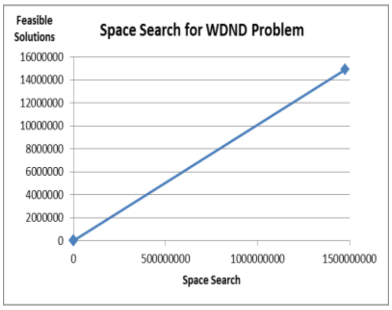


Fig. 2a. Feasible Solutions Space.

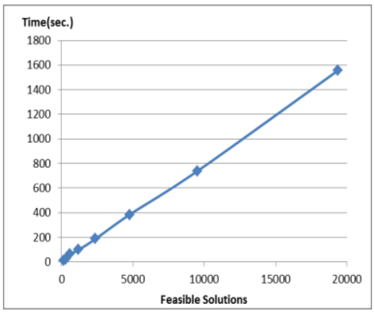
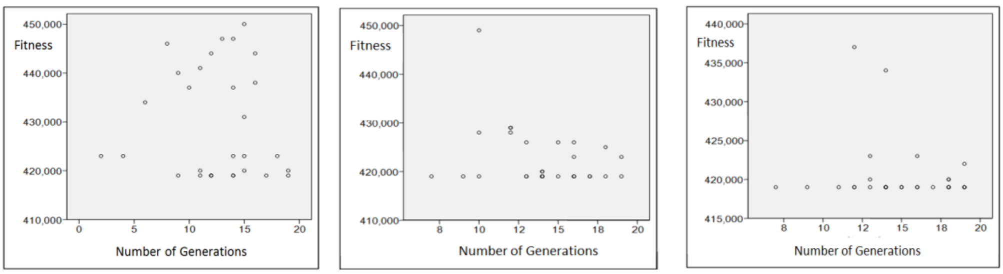
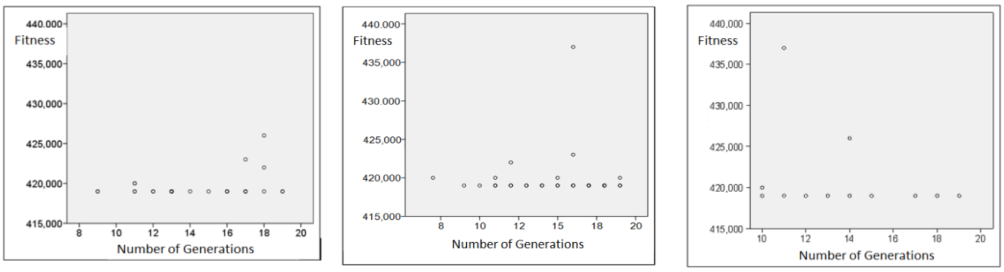


Fig. 2b. Time to Generate Feasible Solutions

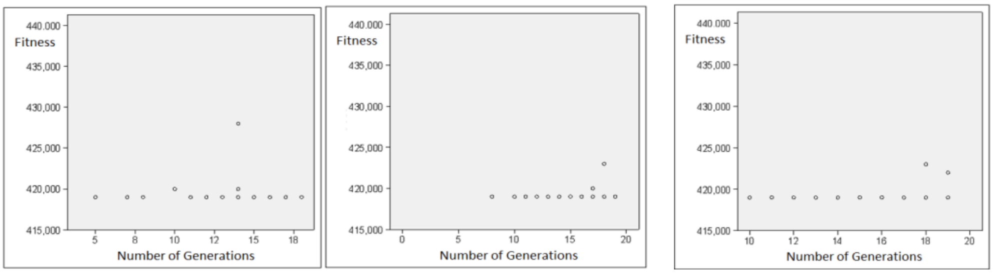
The EA-WDND algorithm was tested using different sized feasible populations. It was executed 30 times for each generated population. On each execution, EA-WDND carried out 20 iterations (generations), labeled 0 to 19. At each generation, the EA-WDND applied the crossover and mutation operators with a probability of 70% and 30% respectively. The population size was kept constant, even when crossover and mutation operators generated more descendants than the population size. Whatever the number of resulting offspring, the population size was the same for all the generations. This was achieved by removing parents and replacing them with the fittest offspring. The offspring were ranked according to their fitness. The best individuals were selected at each generation and they became parents. In some cases, mutations were carried out on them, so they produced new feasible offspring (verified by Epanet) that composed the next generation. It can be said that for each generation the population was created, it was combined to produce feasible offspring (verified in Epanet), and then it was replaced. Table 5 shows the experimental results obtained with EA-WDND after carrying out 30 executions.



100 individuals 200 individuals 300 individuals



400 individuals | 500 individuals | 600 individuals



700 individuals | 800 individuals | 900 individuals

Fig. 3. Best Values Found Using Different Population Sizes

Table 5. Experimental Study of WDND Feasible Solution Space

Population Size	Min. Iteration of Min Cost	Max. Iteration of Min Cost	Min. Cost	Max. Cost	Media Iteration	Media Cost	Number of times
100	9	19	419000	450000	13	429400	9
200	7	19	419000	449000	14	422500	17
300	7	19	419000	437000	15	420566	22
400	9	10	419000	426000	14	419533	25
500	9	19	419000	437000	15	419966	23
600	10	19	419000	437000	15	419866	27
700	5	18	419000	428000	13	419666	26
800	8	19	419000	483000	15	421300	27
900	10	19	419000	423000	16	419233	28

“Max. Iteration of Min Cost” refers to the iteration for which the algorithm, in the worst case, would find the minimum solution for the network cost. For the first row, it means that in the worst case the algorithm would find the minimum solution in iteration 19. “Min. Cost” is the least-cost value for the benchmark. The best cost reported in the literature, for two-looped networks, is 419000. It is the lowest value found in 20 iterations and 30 executions of the algorithm. “Max. Costs” refers to the highest-cost value found in 20 iterations and 30 executions of the algorithm. “Media Iteration” refers to the iterations in which the EA-WDND algorithm finds the best values. It is the average for the iterations of 30 executions. “Media Cost” refers to the average obtained from 30 executions of the algorithm; it is the cost for the network. Number of times refers to the occurrences in which the algorithm finds the best solution. For the first row, it means that the algorithm finds the Min. Cost (419000) in 9 executions. It can be seen that for populations of 900 individuals, the Min. Cost was obtained 28 times. This means that the algorithm failed to find the Min. Cost in only 2 executions, as shown in Table 5. For the best case, the minimum cost was found on iteration number 10, which demonstrates the good convergence of the algorithm.

Also, it can be seen that when working with populations of 700 individuals, the EA-WDND algorithm found the minimum cost for the network, for the best case on iteration number 5 and for the worst case on iteration number 18. It can be seen that the media costs were 419666 and the media iterations was 13.

Figure 3 shows the experimental results obtained by the EA-WDND algorithm. It can be seen that, as the population size increased, better solutions were obtained. Most times, for populations of 900 individuals, the best value known in the literature was obtained. It is important to point out that convergence for this algorithm was reached quite quickly. The best solution known in the literature was found approximately in 80% of the executions, except in the case of populations of 100 individuals.

6. Conclusions and Future works

This paper shows how the performance of a typical evolutionary algorithm can be improved by considering the importance of the population size taken from the feasible solutions space. It shows the experimental results obtained in the solutions space for WDND Problem using a Two-Looped network benchmark. The behavior of the EA is the same as in optimization problems.

According to the obtained results, it can be observed that the feasible solutions space for the WDND problem is 0.01% of the complete solutions space for the benchmark Two-Looped network. For each generation, the population was created, combined to produce offspring, and then died (unfeasible solutions). It was replaced by the best offspring (feasible solutions with Epanet).

It was observed that the removal of parents that had died and their replacement with the fittest offspring helped the EA-WDND converge. It also helped to obtain the best values known in the literature, in iteration number 5 in the best case and iteration number 20 in the worst case. It can be said that the convergence rate and speed was superior for this algorithm.

Continuation of this work includes tests in parallel environments, using larger instances such as the Hanoi and Balerma network.

7. References

[1] R. Baños Navarro, Meta heurísticas Híbridas para Optimización Mono-objetivo y Multi-objetivo, Tesis Doctoral, Almería, Spain, December 2006. Available at www.ace.ual.es/~rbanos/CV.html, last date of access May 12, 2009.

[2] C.H. Papadimitriou, K. Steiglitz, Combinatorial Optimization. Dover Publications, INC, 1998.

[3] M. A. Cruz-Chávez, E. Y. Ávila-Melgar, F. Juárez-Pérez, W. G. Torres-Sánchez, Empirical Transformation of Job Shop Scheduling Problem to the Hydraulic Networks Problem in a Water Distribution System, Electronics, Robotics and Automotive Mechanics Conference, CERMA2009, IEEE-Computer Society, ISBN 978-0-7695-3799-3, pp 76-81, September 22 - 25, México, 2009.

[4] I. Gupta, J.K. Bassin, A. Gupta, P. Khanna, Optimization of Water Distribution System, Environmental Software 8, pp. 101-113, 1993.

[5]. H.D. Sherali, R. Totlani, G.V. Loganathan, Enhanced Lower Bounds for the Global Optimization of Water Distribution Networks, Water Resources Research 34(7), pp. 1831-1841, 1998.

[6] A.L. Costa, J.L. de Medeiros, F.L.P. Pessoa, Global optimization of water distribution networks through a reduced space branch-and-bound search, Water Resources Research, 37(4), p. 1083-1090, 2000.

[7] E. Alperovits, and U. Shamir, Design of Optimal Water Distribution Systems, Water Resources Research, 13(6), 885-900, 1977.

[8] O. Fujiwara, B. Jenchimahakoon, A Modified Linear Programming Gradient Method for Optimal Design of Looped, Water Distribution Networks, Water Resources Research, 23(6), 977-982, 1987.

[9] D.A. Savic, G.A. Walters, Genetic Algorithms for Least-cost Design of Water Distribution Networks, Journal of Water Resources Planning and Management, 123(2), pp. 67-77, 1997.

[10] A. J. Abebe, and D. P. Solomatine, Application of global optimization to the design of pipe networks, in Proceedings of the International Conference on Hydroinformatics, pp. 989– 996, A. A. Balkema, Brookfield, Vt. 1998.

[11] P. Montesinos, A. Garcia-Guzman, J.L. Ayuso, Water Distribution Network Optimization using Modified Genetic Algorithm, Water Resources Research, 35(11), 3467-3473, 1999.

[12] A. Matíaz-Sánchez, Comments on the reliable design of water distribution systems, Revista de obras públicas, No.3.391, 43-49, Octubre 1999.

[13] J. Reca, J. Martinez, Genetic algorithms for the design of looped irrigation water distribution networks, Water Resources Research, Vol. 42, Almería, Spain. 2006.

[14] P.L. Iglesias, F.J. Martínez, V.S. Fuertes, R. Pérez García, Algoritmo genético modificado para diseño de redes de abastecimiento de agua, IV SEREA - Seminario Hispano Brasileño sobre Sistemas de Abastecimiento Urbano de Água, Brasil, 10 de Noviembre 2004.

[15]. S. Liberatore, G.M. Sechi and P. Zuddas, Water Distribution Systems Optimization by Metaheuristic Approach, Advances in Water Supply Management, 265-272, Balkema publisher, Lisse, The Netherlands, 2003.

[16]. P. Montesinos, A. Garcia-Guzman, J.L. Ayuso, Water Distribution Network Optimization using Modified Genetic Algorithm, Water Resources Research , 35(11), 3467-3473, 1999.

[17] [17]. R. Baños, C. Gil, M.G. Montoya, E.Y. Avila-Melgar. Design and Evaluation of Evolutionary Operators for Water Distribution Network Optimisation, Djerba Island Tunisia, October 2010.
[18] B. Adenso-Díaz, H.M. Ghaziri, F. Glover, J. L. González, M. Laguna, P. Moscazo, F. T. Tseng, Optimización Heurística y Redes Neuronales en Dirección de Operaciones e Ingeniería, ISBN 84-283-2269-4, Ed. Paraninfo S. A., Madrid, España, 235p, 1996.
[19] F. Glover. Future paths for integer programming and links to artificial intelligence. Computers and Operations Research, 13:533-549, 1986.
[20] M. A. Cruz-Chávez, A. Rodríguez-León, E. Y. Ávila-Melgar, F. Juárez-Pérez, M. H. Cruz-Rosales, R. Rivera-López, Genetic-Annealing Algorithm in Grid Environment for Scheduling Problems, Communications in Computer and Information Science: Security-Enriched Urban Computing and Smart Grid, Springer Verlag Pub., Berlin Heidelberg, ISSN: 1865-0929, Vol. 78, pp.1-9, 2010.
[21] J. H. Holland, Adaptation in natural and artificial systems. MIT Press, Cambridge, Mass, 1975.
[22] D.E. Goldberg, Genetic algorithms in search, optimization and machine learning. Addison –Wesley Publishing Co., Reading, Mass, 1989.
[23] M. A. Cruz-Chávez, A. Rodriguez-León, E. Y. Ávila-Melgar, F. Juárez-Pérez, J. C. Zavala-Díaz, R. Rivera-López, Parallel Hybrid Evolutionary Algorithm in a Grid Environment for the Job Shop Scheduling Problem, Proceedings of the Second EELA-2 Conference, CIEMAT, ISBN 978-84-7834-627-1, pp 227 - 234, November 25-27, Choroní, 2009.
[24]. R. Baños, C.M. Fonseca, C. Gil, A. Márquez, E.Y. Ávila-Melgar, F.G. Montoya. "Design and Evaluation of Evolutionary Operators for Water Distribution Network Optimisation". International Conference on Metaheuristics and Nature Inspired Computing META´10, Djerba, Tunisia, October 2010
[25]. S. Nazif, M. Karamouz, M. Tabesh, A. Moridi, Pressure Management Model for Urban Water Distribution Networks, Water Resources Management, 2010-02-01Springer Netherlands, 0920-4741.
[26]. C. A. Grosan, Hybrid Evolutionary Algorithms, Series: Studies in Computational Intelligence, Vol. 75, Ajith; Ishibuchi, Hisao (Eds.), XVI, 404 p. 207 illus., Hardcover ISBN: 978-3-540-73296-9, 2007.
[27]. Epanet official site: http://www.epa.gov/nrmrl/wswrd/dw/epanet.html#downloads
[28] Rossman LA (2000) EPANET 2 user's manual. EPA/600/R-00/057, 2000.
[29]. O. Al Jadaan, L. Rajamani, C. R. Rao, Improved Selection Operator for GA , Journal of Theoretical and Applied Information Technology, ISSN 1992-8645, ARPA Press, 269-277, 2008.
[30] G.E. Quindry, E.D. Brill, J.C. Liebman, Optimization of looped water distribution systems. J Environ Eng, ASCE 107(4):665–679, 1981.
[31] I. C. Goulter, B. M. Lussier, and D. R. Morgan, Implications of head loss path choice in the optimization of water distribution networks, Water Resources Research, 22(5), 819– 822, 1986.
[32] A. Kessler, and U. Shamir, Analysis of the linear programming gradient method for optimal design of water supply networks, Water Resources Research, 25(7), 1469–1480, 1989.