Search Algorithm for the Constraint Satisfaction Problem of VRPTW

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Abstract

This paper presents an algorithm called CSP-VRPTW for the Vehicles Routing Problem with Time Windows (VRPTW), which applies the PCP method (Precedence Constraint Posting) used for models of scheduling as a CSP (Constraint Satisfaction Problem). PCP involves the calculation of the shortest path in partial and global form, between pairs of nodes and among all the nodes respectively, in the graph that represents the VRPTW model. In order to apply PCP to VRPTW, the problem is treated as a CSP. The results show that the proposed search algorithm is efficient in the search for the global optimum for some problems.

1. Introduction

The VRPTW is a variant of the VRP (Vehicle Routing Problem) that consists basically of minimizing the transportation cost by restricting the time of each route and the capacity of the vehicle based on the demand of each client [1].

The transport problem works mainly with graph models that can be totally disjunctive [2] and searches for a solution that minimizes the costs of traveling and times based on satisfying restrictions. The Complexity Theory classifies the transport problem as an NP-complete problem [3].

The idea of satisfying restrictions has been used in scheduling problems [4] like Time Tabling [5], Job Shop Scheduling [6] and Fleet Scheduling [7]. Little investigation for the VRPTW problem treated as a CSP (Constraint Satisfaction Problem) exists. Christodoulou [7] et al., mention that when VRP works with Time Windows, the constraint satisfaction technique is competitive with other approaches. Shaw [8] proposes a hybrid algorithm that combines a local search with constraint satisfaction techniques for VRPTW and mentions that his algorithm is competitive with the local search heuristics based on tabu search. These results indicate the existence of an area of opportunity for the creation or improvement of algorithms that work with Vehicles Routing Problems with Time Windows treated as a CSP.

The contribution of this work is the adaptation of the PCP method to the vehicles routing problems with time windows, presenting another alternative, treating VRPTW as a CSP. PCP was designed in order to be applied to the Job Shop Deadline Scheduling Problem (JSDSP) [6],[9], which is formulated as a CSP, based on a tupla {V, D, C}, where V is a variables set, D is a domains set of those variables, and C is a restrictions set of two or more variables in V [1].

In the JSDSP, the goal is to find an optimum sequence for the execution of operations in each one of the machines presented in the problem, respecting constraint precedence between operations that belong to the same job in such a way that it satisfies the deadline for each job. In order to make use of the CSP method, JSDSP is mapped as a VRPTW based on a relaxation of the restrictions of the problem.

Mapping JSSP as a VRPTW:

1. Each operation of the JSSP is a deposit (node) in VRPTW.
2. In JSSP, each job requires the execution of a set of operations in a defined order (precedence constraint). In VRPTW precedence constraints in pairs of nodes do not exist.
3. In JSSP a resource capacity constraint for each machine in the problem exists. The set of all the nodes that form the VRPTW is seen as a machine, for which a single resource capacity constraint exists. This machine is the same as in JSDSP, it is a clique [10] where each node has a path toward the remaining nodes.
4. The processing time of the *k* operation in JSSP represents the distance that exists from the *i* node to the *j* node in VRPTW.

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Section two explains the solution methodology, treating the problem of Vehicles Routing as a CSP, pointing out the adaptation of the constraint satisfaction characteristic of the problem. Section three describes the PCP-VRPTW algorithm highlighting the main functions. Section four shows the experimental results of the PCP-VRPTW algorithm and finally in section five, the conclusions of the carried out work and its main advantages are discussed.

2. Proposed methodology in the application of PCP in VRPTW

The CSP-VRPTW algorithm involves a mixture of techniques to find the shortest path that begins and ends in the deposit (initial node) and passes through all the nodes. It is also a search requirement to find the shortest path between pairs of nodes and the evaluation of each one of the cases that form the PCP procedure. The application of these techniques permits the construction of a solution that gives the number of routes and the number of required vehicles (one per route) as a result, where each required vehicle visits each node of the route that it travels only once and satisfies the demand of the clients based on their capacity. Each vehicle should travel an optimum distance along the route that it corresponds to, respecting the time windows defined in each client (node).

In order to find a solution to the transport problem using the PCP-VRPTW algorithm, the procedure presented in Figure 1 is performed. It takes the information of a benchmark of VRPTW. Based on the Euclidian distances of the problem, a symbolic representation of the model of the disjunctive graph is obtained that represents a VRPTW. It defines the set Ω of the pair of nodes (*i, j*). Next, the algorithm of the shortest path (*sp*) is applied for each pair (*i, j*) of Ω.

With the values of *sp*, the PCP cases of each pair are evaluated (*i, j*) in order to establish the address of each arc for a pair of nodes. The procedure PCP cases are evaluated and some (or all) arcs of the disjunctive graph are transformed into conjunctive arcs. The graph is then verified to see whether the digraph is a feasible solution for VRPTW, that is, if the solution fulfills the established constraints in the definition of the problem. If it is not a digraph and/or it is not a feasible solution, backtracking [11] is done, returning the previous state the nodes that do not have an entrance and exit.

Once there is a solution, the shortest path of this solution is obtained (from the digraph). Using the arcs of the digraph that belong to the shortest path, a greedy algorithm is utilized to divide this route into the number of optimum routes and the number of necessary vehicles (one vehicle for route) in order to satisfy the demand of each client (one client for node).

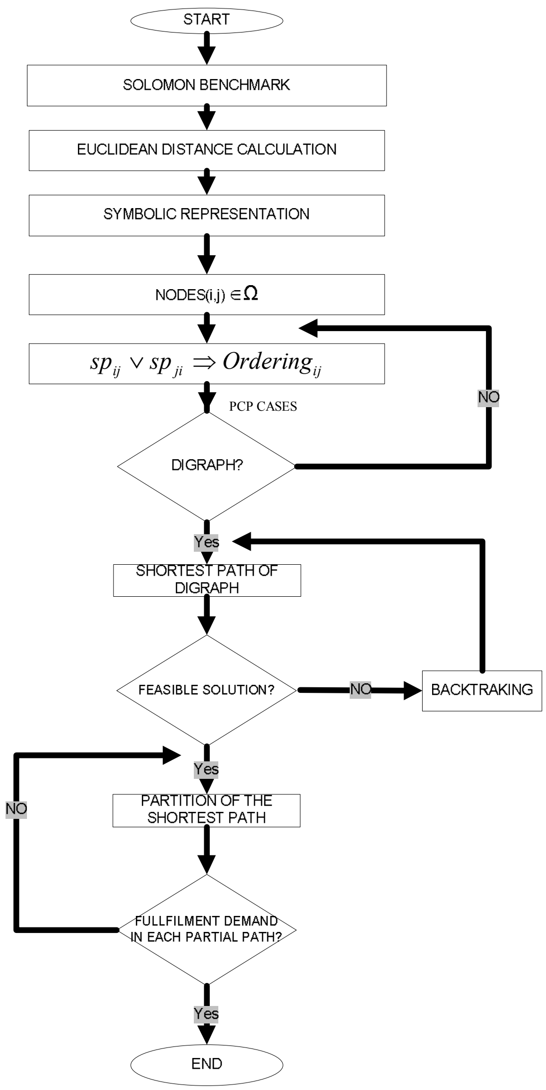

Figure 1. Solution proposal for VRPTW using the PCP-VRPTW algorithm

The VRPTW model with constraint satisfaction expounds satisfy the demand of each client and fulfill the minimum distance of traveling by path.

$$\sum_{i=0}^{n} D_i < ctv \tag{1}$$

$$\sum_{i=0}^{n} R_{ij} = Rtr \tag{2}$$

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In constraints (1), D represents the demand of the clients that should be strictly less than or equal to *ctv*, which represents the total capacity of the vehicle. In restrictions (2), R represents the travel distance between a pair of nodes and should be less than or equal to *Rtr*, which represents the total distance of traveling along a route. In addition to these restrictions, the following also need to be fulfilled:

- The vehicle assignment is one per route.
- The demand for nodes should not exceed the capacity of the vehicle.
- The time established for attention to each client should not exceed the limits of the time window. A time window w_i is defined for the interval [E, L] where E determines the time in which the *i* client hopes to be assisted by a vehicle, and L represents the leaving time of the current *i* node (client) to the following one in the route. $E_i \leq w_i \leq L_i$.

In order to apply the search procedure of CSP, the search of the PCP algorithm is used as a basis. In order to find a solution to the VRPTW model, it is necessary to identify the tupla {V, D, C}. The set V is formed by the $Ordering_{ij}$ variables, where each pair of nodes (*i, j*) defines one of these variables. The set D is the set of values for the variables of V, each $Ordering_{ij}$ taking two possible values, $i \prec j$ or $j \prec i$, first the *i* node is visited and later the *j* node is visited, or vice versa (depending on the defined precedence between *i, j*). The set C of constraints in V is given for each pair (*i, j*) that represents an $Ordering_{ij}$ variable, this set restricts the feasible values for the distance between a pair (*i, j*). PCP defines the set of constraints based on four cases, which are explained in section 3. The search procedure for VRPTW as a CSP consists of six steps:

Step 1. Apply constraints propagation in order to establish the current set V_D of values for each $Ordering_{ij}$ variable not assigned. V_D is obtained by PCP.
Step 2. If V_D = 0 for any $Ordering_{ij}$ variable, backtrack.
Step 3. If there are not variables without assignment or if the assignment is not consistent for all the variables end. Otherwise,
Step 4. Select an $Ordering_{ij}$ variable not assigned.
Step 5. Select a value of V_D in order to assign it an $Ordering_{ij}$ variable.
Step 6. Go to step 1.

3. PCP-VRPTW Algorithm

Within the search procedure, PCP builds the solution through Depth First using partial assignments of Ω. The PCP algorithm carries out a pruning of the search space early on and provides a heuristic for the assignment of values of the $Ordering_{ij}$ variables.

PCP consists of a series of cases in which it should be true that if the shortest path sp between a pair of nodes (*i, j*) that represent the $Ordering_{ij}$ variable then it has a value that fulfills some of the PCP cases. According to the result obtained upon evaluating the shortest path, the value of $Ordering_{ij}$ is designated. The evaluation of *sp* is calculated from *i* to *j* (sp_{ij}) and from *j* to *i* (sp_{ji}).

The PCP algorithm applies the disjunctive graph model of VRPTW. PCP obtains a digraph as a result, and with the help of a greedy algorithm, the number of optimum routes is obtained that satisfies the capacity restriction of each vehicle used in optimum form that represents a feasible solution to the problem.

The PCP-VRPTW algorithm applied to VRPTW, combined with the search procedure PCP for the CSP consists of the following four steps:

Step 1.- Find the shortest path for each unordered pair of nodes sp_{ij} and sp_{ji}.
Step 2.- Classify the decision of ordination of the pairs not ordered with four cases
  Case 1. If $sp_{ij} >= 0$ and $sp_{ji} < 0$ then $i \prec j$ should be selected.
  Case 2. If $sp_{ji} >= 0$ and $sp_{ij} < 0$, then $j \prec i$ should be selected.
  Case 3. If $sp_{ji} < 0$ and $sp_{ij} < 0$, then the partial solution is inconsistent.
  Case 4. If $sp_{ji} >= 0$ and $sp_{ij} >= 0$, then no relationship of order is possible
Step 3.- Existence of cases
  Does either case 1 or case 2 exist?
  If one exists, go to step 4
  If neither exists, go to step 1
Step 4.- Fix new precedence for unordered pairs.

The polynomial that defines the complexity in time of the proposed PCP-VRPTW algorithm for the VRPTW as a CSP is $T(n) = c_1 n^3 + c_2 n^2 + c_3 n + c_4$. The complexity of the proposed algorithm is $O(n^3)$, where *n* is the number of (nodes) clients in the problem.

In order to better understand the algorithm, an example is shown of a small instance of five nodes and a vehicle with a capacity of 200 packages. The disjunctive graph model that is obtained is presented in Figure 2.

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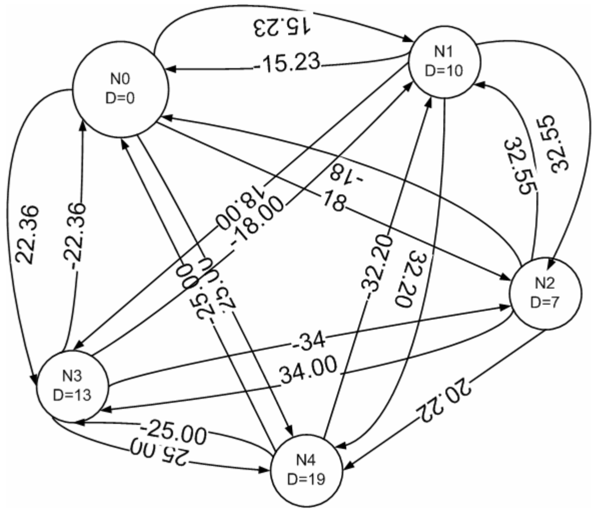

Figure 2. Disjunctive graph

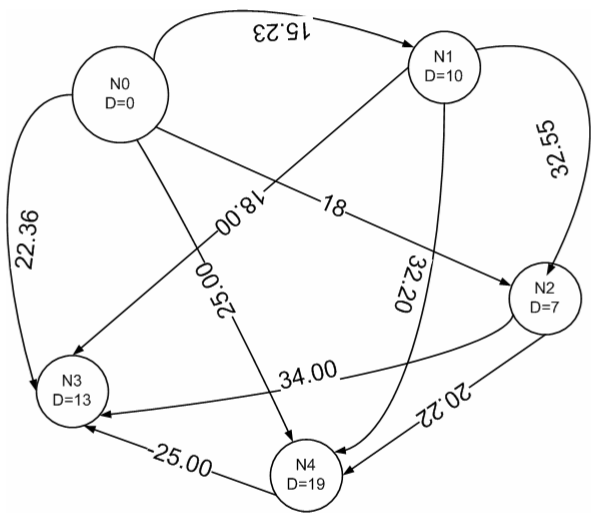

Figure 3. Conjunctive graph

Applying the shorter path algorithm between pairs of nodes and evaluating the PCP cases, the graph shown in Figure 3 is obtained. The resulting graph does not generate a feasible solution, this means that a route from the initial node to the final node does not exist through which each node is passed only once. Because the resulting graph does not generate a solution, backtracking is applied in the nodes with an enter zero and exit zero, leaving fixed the nodes that have at least one entrance and one exit. The PCP-VRPTW algorithm is applied in order to find a route, if a feasible solution is generated, it is taken as a solution. A solution of the problem is shown in Figure 4. Lastly, a greedy algorithm is used which divides the shortest path presented in Figure 4 into a set of routes in order to satisfy the demand constraints of the client and capacity of the vehicles.

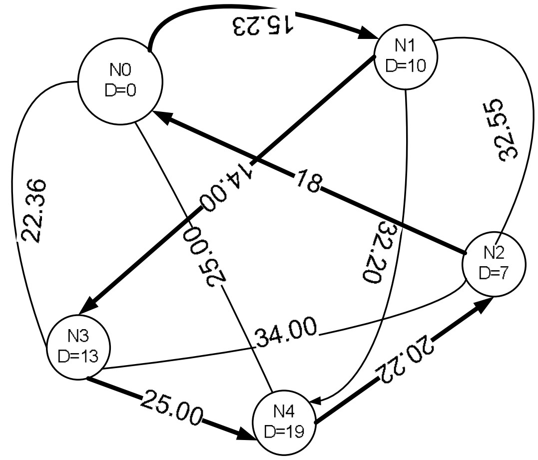

Figure 4. Solution graph

4. Experimental results

The data used for the construction of the solution for the VRPTW was taken from the benchmarks of Solomon [12]. The reported results were generated in a computer with a Pentium processor (R) M with 1.60 GHz and 1GB of RAM using visual C++, in the operating system Windows XP.

Table 1 presents the results generated by the PCP-VRPTW algorithm and the optimum solution reported in the literature for instances of 25 nodes for the benchmarks of Solomon [12]. The best and the worst results were obtained for the execution of the PCP-VRPTW algorithm, one hundred times for each instance. V represents the number of obtained vehicles. Op* and V* are the optimum values reported in the literature for these benchmarks [13], [14], [15]. E_r is the relative error of the solution with regard to the optimum.

Table 1. Comparative results

Bench	V	Best	Average	σ	Op*	V*	E_r
C104	3	193.5	275.4	115.9	186.9	3	3.50
R108	2	411.0	575.9	233.1	397.3	4	3.45
RC108	3	438.5	660.6	314.2	294.5	3	48.9
R208	2	400.6	598.0	279.1	328.2	1	22.1

As can be observed in Table 1, the PCP-VRPTW algorithm obtains good solutions for the instances C104 and R108 which do not surpass an E_r of 3.5. Considering that the PCP-VRPTW algorithm is only a search algorithm used to find a solution and then

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finish, the results found for the first two instances were very near the global optimum and the last two instances were very far from the global optimum. This indicates that the procedure of selective search of PCP which carries out a pruning of the solution space in progressive form is efficient in some problems of VRPTW and not in others. One can also see in Table 1 that for R108, the amount V is half the optimum. This result allows the use of a smaller number of vehicles while increasing by very little the distance (with regard to the optimum) that is necessary to travel only two vehicles.

Figure 5, is a comparison of PCP-VRPTW vs. the reported global optimum, with regard to the total distance traveled by the total units used in each instance. In this figure, it is observed that the approximation of the global optimum with PCP does not depend on the magnitude of the result. For example, the optimum in R108 is of greater magnitude than in R208, and PCP-VRPTW is very near the optimum in R108. On the contrary, in C104 the optimum is of smaller magnitude than in RC108 and PCP-VRPTW is very near the optimum in C104. This could be an indicator that PCP-VRPTW could also give good results in some larger instances with global optimums of greater magnitude.

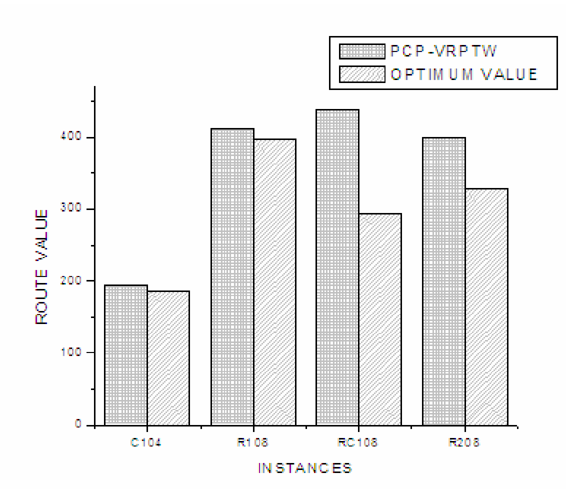

Figure 5. Results of PCP-VRPTW vs. Optimum

5. Conclusions

The main contribution reported in this paper is the first application carried out in the Vehicles Routing Problem with Time Windows with the PCP algorithm, used within a search applied to CSP. In this application, so that PCP could work in VRPTW, an additional step was included which obtained the shortest path of the obtained solution by PCP. This route was divided into several routes in order to satisfy the constraints of the vehicle capacity.

The solutions obtained by PCP-VRPTW are acceptable in half of the revised benchmarks.

However, according to the magnitude of the analyzed optimum values, the possibility exists that for greater instances, PCP-VRPTW could obtain good results in some of these.

It is important to notice that this algorithm carries out a solution search efficiently, delimiting only the solution space of progressive and selective form. This algorithm would be able to improve the operation of metaheuristics like Simulated Annealing and GA if it were applied as part of the search process because PCP-VRPTW conducts a directed search in the solution space.

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