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Mathematical Multi-objective Model for the selection of a portfolio of investment in the Mexican Stock Market

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Abstract

A mathematical multi objective model for the selection of a portfolio of investment is presented and its application in the Mexican Stock Exchange (BMV). The multi objective model proposed is based on our mathematical model of linear programming recently published. Our multi object model is developed whereas the e-constrains method, with which the model remains linear and each iteration the SIMPLEX can be used to determine its solution. Our model is tested with the selection of a portfolio of investment with ten assets of the BMV, where our results are better than those obtained with maximum return and minimal risk models resolved independently.

Keywords: *Multi objective model, Portfolio of investment, Mexican Stock Exchange*

1. Introduction

A portfolio of investment is a combination of assets or individual titles, where this combination of assets to reduce risk and increase the return. From the theoretical point of view, the existence of the risk-return balance is basic for asset evaluation models. While from the practical point of view must have the ability to place the results absolutes in the context of an investment program risk-return features.

The five steps of the process of investment are [1]: The investment policy, values analysis, portfolio construction, portfolio review and periodic evaluation of the return of the portfolio.

In particular for the construction of the portfolio should identify specific actions in which to invest and when to do so. Selectivity, timing and diversification should be dealt with by the investor. Selectivity, also known as micro prognosis, refers to the analysis of values and focuses on the outcome of individual values price movements. The timing, also known as macro forecast, involves the forecast of movements in the price of ordinary assets from the values of fixed income as corporate fertilisers and the letters of the treasure. Diversification is the construction of the portfolio investor that minimizes risk subject to certain restrictions. In this work we focus to make a prognosis micro for an investment portfolio.

The problem of the investment portfolio selection have two objective functions, the first is to maximize the return and the second, no less important than the first, it is to minimize the risk. Two models are essential to the evaluation of a portfolio of investment, Markowitz [2] and the Capital Assets Pricing Model (CAPM) [3]. The first is the quadratic approach for minimization of risk with two restrictions. The first restriction is benefit, whose magnitude change based on a parameter. The second restriction, the sum of the decision variables is equal to 100%. With this model gets the border efficient portfolio risk and return based on the covariance among the assets. The CAPM using a linear approximation, with the same variables to the Markowitz model, gets portfolio investment with a tangent line which touches the border of efficient portfolios.

On the basis of these two models others models have been developed, such as ours [4], where the investment portfolio gets resolved independently features two goals, maximize return and minimize risk, and without considering the covariance among assets. In this model the selected portfolio will be having the minimum between these two problems, solutions difference as establishes CAPM.

On the other hand, models and methods have been developed for selecting portfolio investment as a multi objective problem [5,6,7], they determine the border of efficient portfolios. This border is Pareto front, where not dominated solutions are. To determine these solutions it assumes the search space is convex and its route is through so-called evolutionary algorithms [8], based on different metaheuristics

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On the other hand, models and methods have been developed for selecting portfolio investment as a multi objective problem [5,6,7], they determine the border of efficient portfolios. This border is Pareto front, where not dominated solutions are. To determine these solutions it assumes the search space is convex and its route is through so-called evolutionary algorithms [8], based on different metaheuristics

[5,6,7]. These approaches multi objective, risk and return not always are calculated as arises in the Markowitz model [6], and some other use restrictions as the price of sale [7].

Therefore, this work put develop a linear multi objective model to consider not only the search space of efficient portfolios determined by Markowitz frontier, however the problem arose from the CAPM assumptions that a single point of the tangent line is up to a single point of the aforementioned border.

The second part of this work is a description of the multi objective problems. The foundations of our model are presented in the third. The fourth presents the selection of a portfolio of investment period to the upside.

2. The multi objective problem foundations

The optimization problem multi objective is similar to the problem of global optimization, except for the case multi objective tries to find a vector solution to simultaneously optimize the objective functions, advance knows that these functions are in conflict with each other and improve one means worse performance of the others [8].

The multi objective optimization problem can define mathematically as: To find the vector $\vec{x}^* = \left[x_1^*, x_2^*, \ldots x_n^*\right]^T$ to meet the m inequality constraints:

$$g_i(\vec{x}) \geq 0 \,,\ i = 1, 2, \ldots, m \tag{1}$$

The equality constraints p:

$$h_i(\vec{x}) = 0 \,,\ i = 1, 2, \ldots, p \tag{2}$$

That optimize:

$$\vec{f}(\vec{x}) = \left[f_1(\vec{x}), f_2(\vec{x}) \ldots f_k(\vec{x})\right]^T \tag{3}$$

In other words, it is trying to determine the set of all those numbers that satisfy the restrictions and optimize all objective functions. Constraints define the feasible region of the problem and any $\vec{x}$ vector in this region is considered as a *feasible solution*.

In multi objective optimization, the term optimize changes regarding the optimization (global) mono-objective, because it is to find a target setting among functions instead of a single solution as global optimization. Wilfred Pareto in 1896 gave a more formal definition of the optimum in trouble multi objective [9], which is known today as optimal Pareto. The formal definition is as follows:

Let the set *I*, *I={1, 2, ...},* a vector of decision variables $\vec{x}^* \in F$ (*F* is the feasible region) is a Pareto optimal if there is another $\vec{x} \in F$ such as: $\forall i \in I\left(f_i(\vec{x}) \leq f_i(\vec{x}^*)\right)$ and $\exists j \in I\left(f_j(\vec{x}) < f_j(\vec{x}^*)\right)$. In other words, "the Pareto optimal is the vector of variables which may not improve the objective function problem solutions without worsening anyone other" [8,9]. This result in a set of solutions called *Pareto optimal set*. The set of vectors that correspond to a solution, included in the set of optimal Pareto, are called not dominated.

2.1. Pareto optimal set

For a multi objective problem $\vec{f}(x)$, the Pareto optimal set (P^*) is defined:

$$P^* = \left\{x \in F : \neg\exists x' \in F \quad \vec{f}(x') \precsim \vec{f}(x)\right\} \tag{4}$$

2.2. Pareto Dominance

Pareto dominance can be defined by: A vector $\vec{u} = (u_1, \ldots, u_k)$ domination another $\vec{v} = (v_1, \ldots, v_k)$ (through $\vec{u} \precsim \vec{v}$) If and only if *u* is partially below *v*:

$$\forall i \in \{1,\ldots,k\}, u_i \leq v_i \wedge \exists i \in \{1,\ldots,k\} : u_i < v_i \tag{5}$$

So solution domination to another, the solution needs to be strictly better in at least one objective and not worse in any of them.

2.3. Pareto Front

The representation of functions whose vectors are not dominated, and are in the set of Pareto optimal is called Pareto front. Its definition is as follows: For a process problem $\vec{f}(x)$ and a set of Pareto optimal P^*, the Pareto front (FP^*) is defined by:

$$FP^* = \left\{ \bar{f} = [f_1(x),\ldots,f_k(x)] : x \in P^* \right\} \tag{6}$$

The best way to find the Pareto front is evaluate all and each point in the feasible region. The exhaustive search does not computable problem. As such, it is the need to apply or developing heuristics to produce the Pareto front approaches. Figure 1 describes each element defined previously.

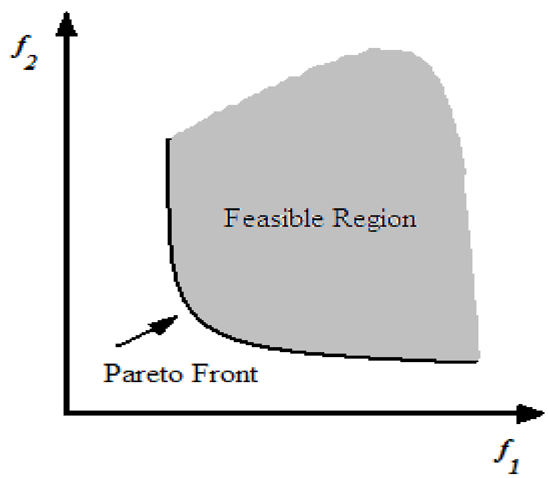

Figure 1. Front of Pareto with two objective functions.

2.4. Brief description of the multi objective solution algorithms

To resolve the multi objective problems, a vector of solutions, has developed the evolutionary computation. Evolutionary computation consists of a set of heuristics based its operation on the mechanism of natural selection proposed by Charles Darwin, and then extended in the so-called Neo-Darwinism [8].

In evolutionary computation a population is composed of individuals, an individual is a solution to a problem and is encoded according to the needs of the problem, for example in a string of bits. The medium where develops this individual is represented by the objective function, while the problem restrictions indicate so fit what the individual to survive in that environment. Individuals of the population, called parents, apply probabilistic, crosses and mutation operators keep some properties from their ancestors for new individuals called children. The ancestors are retained or removed by a deterministic or probabilistic selection; this is done with individuals of the population to form a new population with new individuals. This process is repeated over a number of iterations called generations in evolutionary computation.

With linear programming multi objective are three methods to resolve such problems, which are: weightings, e-constrain and multi objective SIMPLEX [10]. The first two are also used for non-linear problems while the latter is only for linear problems. Each has its application, which we use to select the appropriate method for our problem.

2.4.1. Weightings Method

To carry it out must be systematically consider a range of positive weights multiplying each objective problem sets. Usually begins with individual optimization of each objective, it is to consider the weights *{1, 0,..., 0}, {0, 1,..., 0}, {0, 0,..., 1}*. Later it is introduced a systematic variation of these weights with a preset rate of increase. Each issue of weighting is a problem of linear programming, which leads to an efficient solution. It can have certain disadvantages: different sets of weights can generate the same point, the size of a set of weights step to another may not allow generate extreme points and, therefore, closer to the efficient set would be obtained.

2.4.2. E-constrains Method

It is to optimize the objective function more important than the others, where they will be limited by the lower dimensions. Lower dimensions represent subjective preferences of the decision-maker, so if there is not a solution should be relaxing at one of the dimensions. The solution of the problem will be efficient if it is a single solution.

2.4.3. SIMPLEX-Multi objective method

This method is a natural extension of the SIMPLEX algorithm and is composed of three stages, as it uses the same Pivot transformation to move from an efficient endpoint to another adjacent. The three stages are:

1. Determine an initial basic solution possible. This is carried out, as the one objective, introducing artificial slack variable, which results in a possible initial endpoint.
2. Determine an efficient endpoint whose existence is guaranteed. If the feasible region of the problem is non-empty and all objective functions are bounded on it, then there exists at least one efficient endpoint.
3. Finally, efficient points are determined from the resolution of the previous stage, and the remaining extreme points are generated from that same solution.

3. Fundamentals of our linear multi objective algorithm

There are two assumptions made for our algorithm, which is described below:

1. Assume that the portfolios investment with maximum return and minimum risk is online of portfolios efficient; it is in front of Pareto. This portfolio is in equilibrium because is not dominated solution.
2. The determination of the optimum portfolio is possible with the CAPM, which sets one portfolio investment is determined from the point where touch the straight line tangent to the curve efficient portfolios.

As a result, the problem is to calculate a single point of Pareto front; the point is determined by the CAPM. From this observation, we develop our multi objective mathematical model. The Figure 2 describes the graphical interaction of the two models, the Markowitz and the CAPM. In the Figure 2 the **FF** line represents the Pareto front optimal investment portfolios. The **SS** line is the line security market, where the R_f point represents the risk-free return. The point **a** on the **FF** line represents investment portfolio optimal return R and a risk σ . As a result, the **a** point is the point that has the smallest difference between risk and return [3].

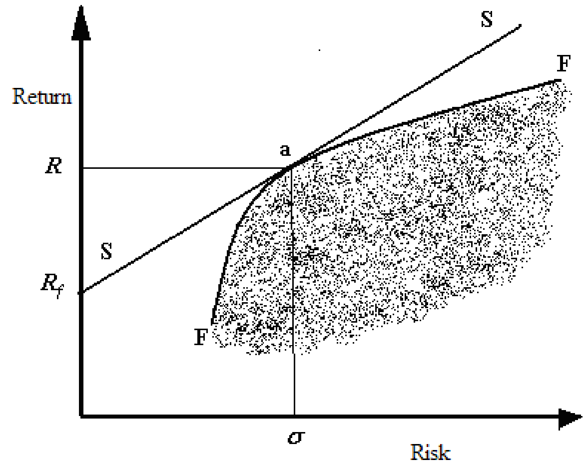

Figure 2. The CAPM

The graph of Figure 3 is made by the selection of a portfolio of ten actions Mexican stock exchange (BMV), investment results during July 2005 to February 2007 [11]. The graphic shows that the determination of the point **a** is from the region of feasible solutions.

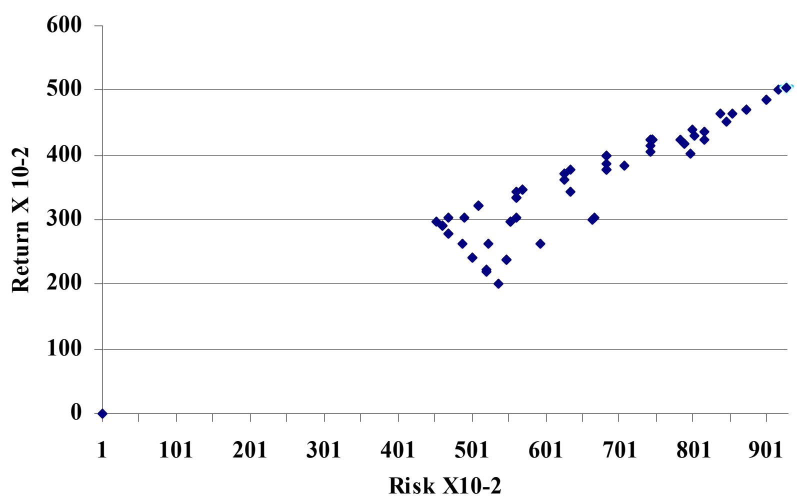

Figure 3. During the July 2005 to February 2007 investment portfolios feasible region [11]

Figure 3 shows that only a point is the solution, this is the one more to the left of all the points, and it has the minimal difference between risk and return.

Due to the two problems are resolved independently, the maximize return and minimize risk, in figures 4 and 5 are plotted return and risk based on the minimal difference between them, respectively The figures 4 and 5 graphs show are convex areas, as expected, since when a problem has a solution by means of a linear programming model cannot exist surface non-convex [10].

Within the sets of items that are plotted in figures 4 and 5 is the element with the smallest difference between return and the risk, point **a** of the figures 2 and 3. In other words, with elements of these graphics, independently, determines the point **a**, with which is: $\exists i \in \{1,2,...,k\} : R_i \wedge \sigma_i \rightarrow \mathrm{a}$

As a result of the above, the determination of the point **a** can do through the search R_i return and risk σ_i (Figures 4 and 5). This search can be done through changes in the magnitude of the restrictions to reduce the search space, and assess the "fitness" as the minimal difference between risk and return. The above is based on the e-constrain method.

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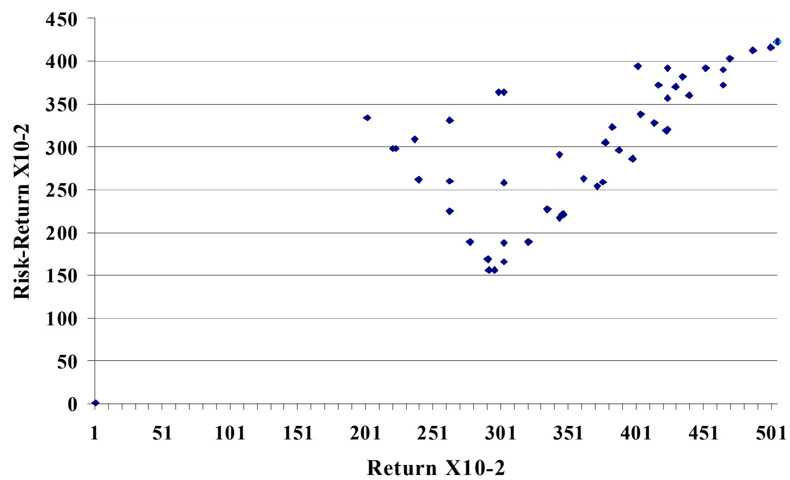

Figure 4. Performance based on the minimal difference for 10 assets, July 2005 to February 2007 period [11].

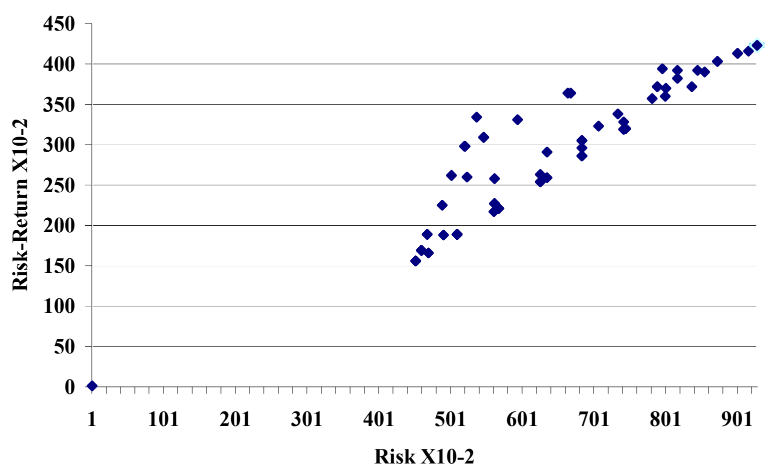

Figure 5. Risk based on the minimal difference for 10 assets, July 2005 to February 2007 period [11]

4. The linear multi objective problem

The following briefly describes assumptions, maximize return and minimize risk, the elaboration of the process model models. [4]:

First assumption. The minimum required amount of money is the cost of assets more cheaply, from this amount reaches the amount of money the most expensive assets.

Second assumption. The minimal risk portfolio of investment is the risk of the asset that has the lowest of all assets, and it will vary to higher risk of the appropriate asset.

Third assumption. Minimum return of the investment portfolio is the return of the asset that has lowest return of all titles, and it will vary to the highest performance of the appropriate asset.

With these assumptions were made [4] following models:

4.1 Model to maximize return

$$\text{Maximize } z_1 = \sum_{i=1}^{n} x_i R_i \tag{7}$$

Subject to:

$$\sum_{i=1}^{n} x_i \left(Pv_i - Pv_{\min}\right) \leq \lambda_1 \left(Pv_{\max} - Pv_{\min}\right)$$

$$\sum_{i=1}^{n} x_i \left(\sigma_i - \sigma_{\min}\right) \leq \lambda_2 \left(\sigma_{\max} - \sigma_{\min}\right)$$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \leq 1$$

Where R_i, σ_i and Pv_i are return, risk and selling price of action i, respectively. x_i is title i you must purchase and is a real variable $0 \leq x_i \leq 1$. $x_i = 0$ when the action is not part of the investment portfolio. λ_1, λ_2: are variable used to iterate through all the space of solutions, and their values are $0 \leq \lambda_1, \lambda_2 \leq 1$.

4.2. Minimization of the risk model

$$\text{Minimize } z_2 = \sum_{i=1}^{n} x_i \sigma_i \tag{8}$$

Subject to:

$$\sum_{i=1}^{n} x_i \left(Pv_i - Pv_{\min}\right) \geq \lambda_1 \left(Pv_{\max} - Pv_{\min}\right)$$

$$\sum_{i=1}^{n} x_i \left(R_i - R_{\min}\right) \geq \lambda_2 \left(R_{\max} - R_{\min}\right)$$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \leq 1$$

The variables are the same as in the problem of maximizing of 4.1.

4.3 Linear multi objective model for the selection of an investment portfolio

Whereas the two models (7) and (8), the e-constrains method and assumptions referred to in point 3, the resulting multi objective model is shown below:

$$\text{Minimize } z_3 = \sum_{i=1}^{n} x_i \left(\sigma_i - R_i\right) \tag{9}$$

Subject to:

$$\sum_{i=1}^{n} x_i \sigma_i = \sigma_{\min} + \lambda_1 \left(\sigma_{\max} - \sigma_{\min}\right) \tag{10}$$

$$\sum_{i=1}^{n} x_i R_i = R_{\min} + \lambda_2 \left(R_{\max} - R_{\min}\right) \tag{11}$$

$$\sum_{i=1}^{n} x_i Pv_i \geq Pv_{\min} \tag{12}$$

$$\sum_{i=1}^{n} x_i = 1, \quad x_i \leq 1$$

In this model the variables are the same of 4.1.

Restrictions (10) and (11) are the objective features two models of optimization base, whose magnitude will vary from the lower bound to the upper bound, as sets it the e-constrains method. In addition, they reflect the second and third assumption of the original approach. The restriction (12) reflects the first assumption of original models.

Because restrictions (10), (11) and (12) are valid for elements of the set of feasible region, first thing it does is to identify at least one element of that region. Once already has an element of the feasible region makes the journey to the element which is the optimum portfolio. Figure 6 shows a schematic route strategy to determine the optimal investment portfolio.

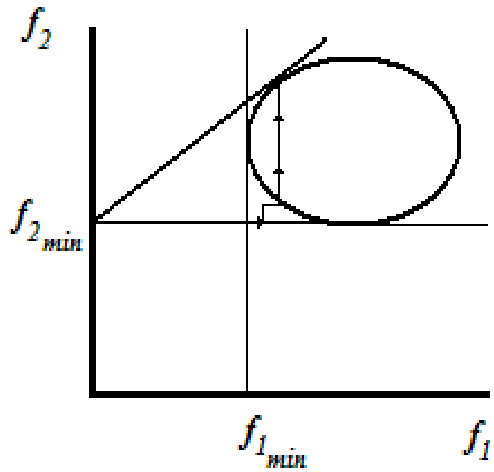

Figure 6. The multi objective model search strategy.

The search strategy is to start from the origin produced by minimum dimensions (f_{1min} and f_{2min}). Subsequently f_1 and f_2 increase, first the f_1 function and then the f_2, and so on until reach a workable solution. Once came to a workable solution, runs the convex region while maintaining f_1 constant up to the front of Pareto. When the Pareto front is reached, $\Delta\lambda$ is modificated; it is split into two that increase each time. The first modification will be $\frac{1}{2}\Delta\lambda$, the second $\frac{1}{4}\Delta\lambda$ and so on up to $\frac{1}{2^m}\Delta\lambda$, where m is the number of modifications required to reach the optimal considered portfolio.

5. Application of the multi objective model to calculate a Mexican stock investment portfolio.

It are chosen 10 assets more negotiated companies of the Internet of the Mexican Stock Market (BMV) page, assets considered for the calculation of the CPI [12], during the period from July 2005 to February 2007. The return and the risk values were obtained as required by the models described previously, equations (13) and (14) [1].

$$R_i = \frac{\sum_{j=1}^{m} r_{i_j}}{m} \tag{13}$$

$$\sigma_i = \sqrt{\frac{\sum_{j=1}^{m}\left(r_{i_j} - R_i\right)^2}{m-1}} \tag{14}$$

Where R_i is the average return of asset i, r_i is the return of the asset i in each period j and m is the number of periods concerned. σ_i is the risk (return standard deviation). In the table 1 are the 10 assets risks and returns.

In table 2 are the results obtained by the process model and strategy of search designed for this problem. In that same table are also results from the same problem with the models mono-objective maximize return and minimize risk.

Table 1. Return, price and risk of ten companies during the period from July 2005 to February 2007 [11,12].

Var.	Titles	R_i (%)	Pv_i	σ_i (%)
x_1	GMEXICOB	5.053	76.00	9.289
x_2	WALMEX V	2.230	39.67	5.985
x_3	G MODELOC	2.016	59.11	5.381
x_4	CEMEX CPO	1.024	35.47	10.331
x_5	TELMEX L	1.834	18.85	6.332
x_6	TELEVISACPO	2.388	55.56	6.569
x_7	URBI	4.147	46.42	7.851
x_8	KIMBERA	1.135	43.52	6.191
x_9	TELECOMA1	3.140	49.18	9.459
x_{10}	NAFTRAC02	2.962	30.82	4.527

Table 2. Results of the multi objective model (julio2005-February 2007)

Model		x_1 (%)	X_3 (%)	x_5 (%)	x_7 (%)	x_{10} (%)	Sum (%)	Dif (%)
Minimize Risk	(%)	1.05	36.82	0.0	0.0	62.13	100	
	R_i	0.053	0.742	0.0	0.0	1.840	2.636	2.256
	σ_i	0.098	1.981	0.0	0.0	2.813	4.891	
Maximize Return	(%)	0.0	0.0	0.0	69.81	30.19	100	
	R_i	0.0	0.0	0.0	2.895	0.894	3.789	3.058
	σ_i	0.0	0.0	0.0	5.481	1.367	6.847	
Multi objective	(%)	8.56	0.0	0.02	0.0	91.42	100	
	R_i	.4325	0.0	.0004	0.0	2.7078	**3.1407**	1.794
	σ_i	.7951	0.0	.0013	0.0	4.1385	**4.9349**	

As shown by the results of table 2, the return rate determined by the multi objective model is slightly more close to the determined by the model to minimize the risk, that the calculated value is slightly lower than the return of the arithmetic average of the two models (3.2125%). While the risk determined by the multi objective model is slightly higher up the minimization of the risk model. The solution obtained is, closer to those best determined by the minimization of the risk model, but with a much higher rate of return.

Best results due to increases in the parameters were smaller than rates used by the mono objectives models. Mono objective models used a constant increase of 0.1 to traverse the whole search, while in the multi objective model came to use $0.1/2^5$ increments in recent iterations. This allows us to obtain a better solution. 16 Iterations and 5 increases required to get the results presented in table 2.

6. Conclusions

It is concluded that the multi objective approach to the selection of an investment portfolio is better than independently troubleshoot maximize return and minimize the risk.

The second conclusion is that the CAPM assumptions can be applied to the process problem and with them, it is possible to limit the search space to a single point and not determine all front Pareto.

The third conclusion is that the results show that the search can be by means of lower dimensions changing and do not necessarily require an evolutionary algorithm for the generation of individuals who can form a solution.

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