Artificial Neural Networks for Inverse Heat Transfer Problems

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Abstract

We present a solution for an inverse heat transfer problem involving internal heat generation using artificial neural networks. The problem involves a heat conduction problem with internal heat source in cylindrical coordinates. The network is a feedforward with backpropagation algorithm. We compare the results with the Levenberg-Marquardt Method and discuss advantages and disadvantages. The two methods recover very well the optimum parameters.

1. Introduction

Accordingly to Oleg Mikhailovich Alifanov [1], one of the great proponents of inverse problems: Solution of an inverse problem entails determining unknown *causes* based on observation of their *effects*. This is in contrast to the corresponding direct problem, whose solution involves finding effects based on a complete description of their causes.

Here, according to the accepted methodology, we mean by *causal characteristics* of heat transfer in the body or in the system a boundary conditions and their parameters, initial conditions, thermophysical properties, internal sources of heat and conductivities as well as geometric characteristics of the body or the system. Then the *effect* is a heat state which is determined by the temperature field of an object studied.

In this paper we present an application of Artificial Neural Networks (ANN) in the solution of an Inverse Heat Transfer Problem (IHTP). There are many applications [7][8][10] but nonetheless recover the heat source generation. First, we present the direct problem related with the inverse problem. Then, we show the steps for the solution. Finally, we compare the results and conclusions are made.

2. Direct Problem

The guarded hot plate apparatus is generally recognized as the primary absolute method for measurement of the thermal transmission properties of homogeneous insulation materials in the form of flat slabs. This test method has been standardized as ASTM Test Method (C 177)[2] and ISO International Standard (IS0 8302) [6], with the two test methods being very similar, but not identical. This test apparatus realizes the laboratory measurement of the steady-state heat flux through flat, homogeneous specimen when their surfaces are in contact with solid, parallel boundaries held at constant temperatures. In this kind of device, an electrical current is applied inside the hot plate and inside the guard for obtaining a constant temperature. Experimental test for guard and hot plate involves measurement of temperature until steady state is held. We use this test to analyze the ANN performance for solving inverse heat transfer problems.

The mathematical model [3] for the hot plate is given by

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{g \cdot \delta\left(r - r_1\right)}{2\pi k r} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

with boundary condition in $r = b$

$$k\frac{\partial T}{\partial r} + hT = hT_a \tag{2}$$

and initial condition

$$T = T_0 \qquad 0 \leq r \leq b \tag{3}$$

The solution [9] for this problem is given by

$$\begin{aligned} T(r,t) &= \frac{hbT_0}{k\beta^2 N} R_0\left(\beta, r\right) R_0\left(\beta, b\right) e^{-\alpha\beta^2 t} \\ &+ \frac{hbT_a}{k\beta^2 N} R_0\left(\beta, r\right) R_0\left(\beta, b\right)\left(1 - e^{-\alpha\beta^2 t}\right) \\ &+ \frac{g}{2\pi k\beta^2 N} R_0\left(\beta, r\right) R_0\left(\beta, r_1\right)\left(1 - e^{-\alpha\beta^2 t}\right) \end{aligned} \tag{4}$$

where $R_0\left(\beta, r\right) = J_0\left(\beta r\right)$ are eigenfunctions for the next eigenvalue problem

$$\beta k J_1\left(\beta b\right) - hJ_0\left(\beta b\right) = 0 \tag{5}$$

and N is the norm given by

$$N = \frac{b^2\left(h^2 + \beta^2 k^2\right)}{2k^2\beta^2} R_0^2\left(\beta, b\right) \tag{6}$$

The heat generation function g is supposed to be constant. Therefore, it can be expressed as

$$g = P_1 \tag{7}$$

We can simulate different input conditions with this solution to train the ANN.

3. Inverse Problem

An inverse solution can be understood [10] as an attempt to find out the inverse operator P^{-1} (or an approximation G for it):

$$P[g(t)] = T(r,t) \quad \Rightarrow \quad g(t) = P^{-1}[T(r,t)] \tag{8}$$

A typical approach to compute the unknown $g(t)$ is to formulate the inverse problem as a non-linear optimization problem $\min S(g)$ where:

$$S(g) = \|T^{exp} - T^{mod}(g)\|^2 + \alpha^* \Omega[g(t)] \tag{9}$$

being T^{exp} measured quantities, T^{mod} are computed quantities from a mathematical model, and Ω is a regularization operator. The approach based on the artificial neural networks is to design a non-linear mapping to obtain an approximated inverse solution: $g(t) = G_{NN}[T(r,t)]$, where $G_{NN} \sim P^{-1}$.

4. Artificial Neural Network Model

An artificial NN is an arrangement of units characterized by: a large number of very simple neuron-like processing elements; a large number if weighted connections between these elements, where the knowledge of the network is stored; highly parallel, distributed processing. The processing element in an ANN is a linear combiner with multiple weighted inputs, followed by an activation function.

ANN have two stages in their application, which are the learning and activation steps. During the learning step, the weights and bias corresponding to each connection are adjusted to some reference examples. For activation, the output is obtained based on the weights and bias computed in the learning phase (Fig. 1). There are many possible arrangements and learning strategies. For the present inverse problem, the neural network architecture implemented is a multilayer perceptron (MP) with backpropagation algorithm. See references [4, 5] for a full description of these ANN architectures.

The MP-NN has one input layer, one or more hidden layers, and one output layer. It is a feed-forward network and employs a back-propagation algorithm for the learning process.

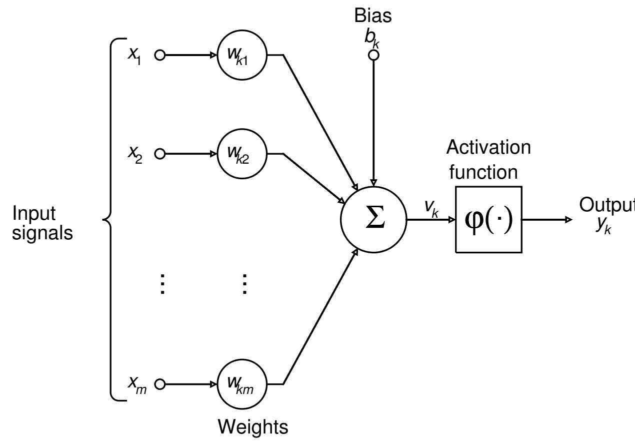


Figure 1. Neural Network Model

5. Simulations

For the ANN, the training sets are constituted by synthetic data obtained from the forward model, i.e., time-series for a measure point close to the boundary ($r = 0.0762$ m). The training step data set is the time-series obtained from

$$g = 0.5n \tag{10}$$

with $n = 1, \ldots, 19$. The validation step data set is obtained from

$$g = \frac{n\pi}{3} \tag{11}$$

with $n = 1, \ldots, 4$. And we include a test step data obtained from

$$g = \frac{ne}{3} \tag{12}$$

with $n = 1, \ldots, 4$. For all data sets a random noise generated by

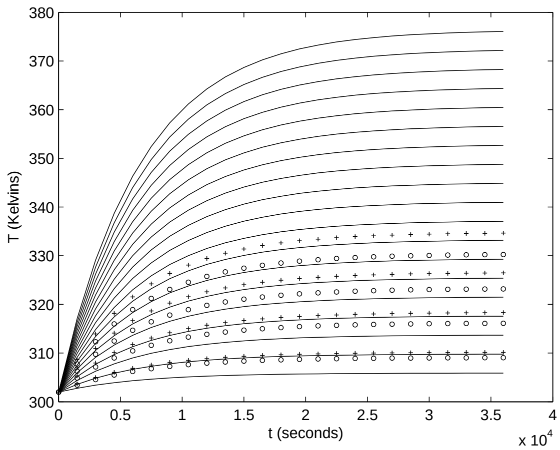
$$T_{\text{perturbed}} = T_{\text{analytical}} + \cos(rand * \pi) \tag{13}$$

was added, simulating the real experimental data. Fig. 2 show temperature field for both analytical and perturbed data sets in every step.

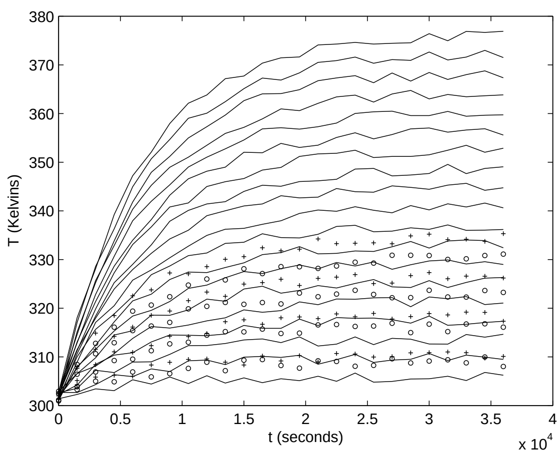
The ANN was trained with 25 inputs (a time step of 1500 seconds). Three neurons with hyperbolic tangent sigmoid transfer function were use in the hidden layer and just one neuron with linear transfer function in the output layer (Fig. 3). In this work, the Levenberg-Marquardt training algorithm –in the Matlab Neural Network Toolbox [4]– was used.

6. Results

Simulation with experimental and perturbed data was made and the correlation coefficient between the outputs



(a) Analytical



(b) Perturbed

Figure 2. Temperature distributions. Training(-). Validation(+). Test (○)

and the targets are shown in Table 1. It is a measure of how well the variation in the output is explained by the targets. If this number is equal to 1, then there is a perfect correlation between targets and outputs. In the three data sets, the number is close to 1, which indicates a good fit.

In Fig. 4 the network outputs are plotted versus the targets as open circles. The best linear fit is indicated by a dash line. The perfect fit (output equal to target) is indicated by the solid line. For training, it is difficult to distinguish the best linear fit line from the perfect fit line because the fit is so good. For validation, there is a little bit difference. But for test, the difference increase and it is notorious.

In the experimental test for hot plate a 5 W heat source was induced. Measurements was taken and used with ANN

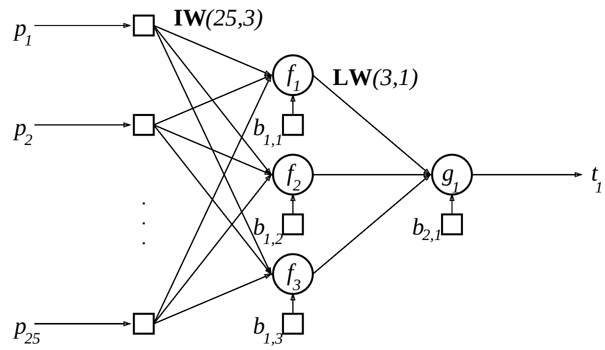


Figure 3. Neural Network used for simulation

Table 1. Correlation coefficient for simulation of training, validation and testing data sets.

Type	r
training	0.99904909335456
validation	0.999771262730603
test	0.985638935321031

model. The results are shown in Table 2. It shows the comparison with Levenberg-Marquardt method [3]. From the results it is evident that ANN improves the results obtained with Levenberg-Marquardt method.

7. Conclusions

A neural network model was developed for solving inverse problems in heat conduction. Results with training, validation and test data sets were shown for performance analysis of the neural network. The neural network was applied to recover the heat generation function g of the hot plate in a Guarded Hot Plate Apparatus. Results were improved respect to Levenberg-Marquardt method.

Artificial neural networks are a good tool for solving inverse problems. There is a combination of neural networks with other methods such as Levenberg-Marquardt method [11] for improving results.

Table 2. Comparison of the Artificial Neural Network (ANN) model and Levenberg-Marquardt (LM) method.

Model	rse
ANN	0.000349679638717
LM	0.000397264805109

References

[1] O. M. Alifanov. *Inverse Heat Transfer Problems*. International series in heat and mass transfer. Springer-Verlag, 1994.

[2] ASTM. *Thermal Insulation; Enviromental Acoustics*, volume 04.06 of *2004 Annual Book of ASTM Standards*, chapter C177-04 Standard Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus. American Society for Testing and Materials, Philadelphia, 2004.

[3] O. Cortés. Aplicación del método de levenberg-marquardt y del gradiente conjugado en la estimación de la generación de calor de un aparato de placa caliente con guarda. Master's thesis, CENIDET, 2004.

[4] H. Demuth, M. Beale, and M. Hagan. *Neural Network Toolbox 5: User's Guide*. The Mathworks, Inc., 3 Apple Hill Drive Natick, MA 01760-2098, 2007.

[5] S. Haykin. *Neural Networks. A Comprehensive Foundation*. Prentice Hall, New Jersey, USA, 1999.

[6] ISO. *Energy and heat transfer engineering; Heat recovery. Thermal insulation*, volume 27.220, chapter 8302:1991 Thermal insulation – Determination of steady-state thermal resistance and related properties – Guarded hot plate apparatus. International Organization for Standardization, Switzerland, 1991.

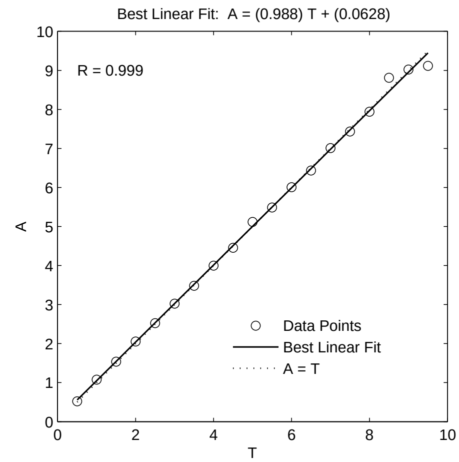
[7] E. Issamoto, F. T. Mikki, J. I. da Luz, J. D. da Silva, P. B. de Oliveira, and H. F. C. Velho. An inverse initial condition in heat conduction: A neural network approach. In *Brazilian Congress on Mechanical Engineering*, Unicamp, Campinas (SP), Brazil, November 22-26 1999.

[8] F. T. Mikki, E. Issamoto, J. I. da Luz, P. P. B. de Oliveira, H. F. Campos-Velho, and J. D. S. da Silva. A neural network approach in a backward heat conduction problem. In *Proceedings of the IV Brazilian Conference on Neural Networks*, pages 19–24, ITA, Saõ José dos Campos (SP), Brazil, July 20-22 1999.

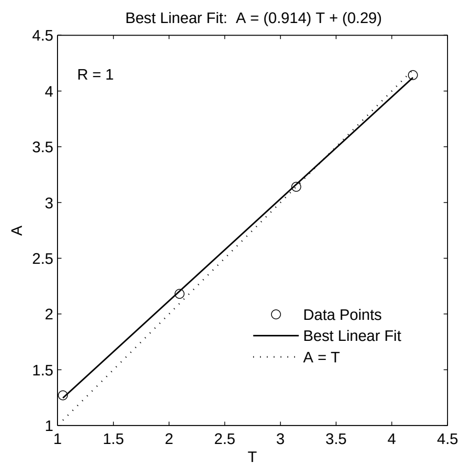
[9] M. N. Özisik. *Boundary Value Problems of Heat Conduction*.

[10] E. H. Shiguemori, F. P. Harter, H. F. C. Velho, and J. D. S. da Silva. Estimation of boundary conditions in conduction heat transfer by neural networks. *Tendências em Matemática Aplicada e Computacional*, 3(2):189–195, 2002.

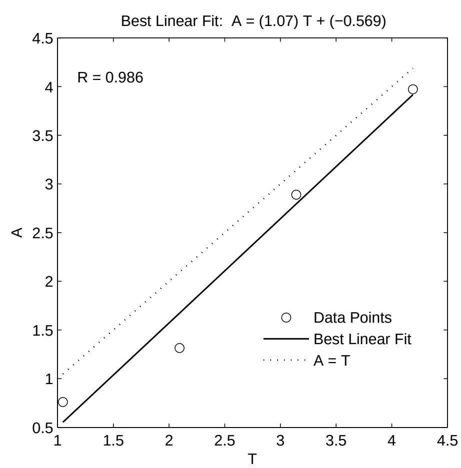
[11] F. J. C. P. Soeiro, H. P. C. Velho, and A. J. S. Neto. A combination of artificial neural networks and the levenberg-marquardt method for the solution of inverse heat conduction problems. In *Anais do VI Encontro de Modelagem Computacional*, Nova Friburgo, 2003.



(a) Training



(b) Validation



(c) Test

Figure 4. Regression analysis between the network response and the corresponding targets