Empirical Transformation of Job Shop Scheduling Problem to the Hydraulic Networks Problem in a Water Distribution System

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In this paper an analogy of the Job Shop Scheduling Problem to the Hydraulic Networks Problem is presented by mapping this model of scheduling, using as a base the disjunctive graph model. The mapping carried out allows visualization of the Hydraulic Networks problem as an NP-complete model with constraints defined in the Job Shop Scheduling Problem. The mapping presented indicates that the Hydraulic Networks Problem is a difficult problem to solve by using an approach with the constraints of an NP-complete problem.

1. Introduction

The water distribution networks design belongs to a wide group of problems. Finding the optimum solution to these problems is extremely complex, even sometimes impossible [1]. These problems have been thoroughly studied in the last few decades, by diverse authors. In [2] the authors propose several methods and techniques in order to deal with this type of problem using theoretical models. But when attempts are made to solve these problems for real instances, it becomes increasingly complex to find the optimum solution.

The water distribution problem is classified as a complex optimization NP-hard problem [3]. Basically, it consists of finding the most efficient way to give water to users, within given constraints.

Several approaches for the water distribution problem in hydraulic networks have been proposed. Global optimization [4, 5] and linear [6, 7, 8, 14] and non-linear programming [9] have been applied. In addition, many heuristics have been used in order to solve this problem [1, 2, 3, 10, 11]. To date, no information was found that proposed a transformation of the Job Shop Scheduling Problem into the Hydraulic Networks Problem, to take advantage of the immense information available on applied techniques related to the Job Shop Scheduling problem

Among the most important restrictions of the hydraulic networks problem is the quality of service at a low distribution cost. The quality of service refers to fulfilling the minimum pressure requirements of the hydraulic network users. The low distribution cost seeks the most economic solution possible. Service time for the users is also considered, so that each and every one of the users can be assisted at any moment.

The hydraulic network consists of various elements including: the supply sources, which are the deposits which offer service to users; nodes that represent the users of the deposits, such as people, companies, cultivation areas and others; and tubing, which establishes a connection between deposits and users by allowing water to be delivered to the network users. The type of topography present is important, whether there are ramified or mesh type networks [11].

For ramified networks, it is possible find the best solution using mathematical methods and exact algorithms, as seen in [12] where maximum flow and maximum flow at minimum cost are proposed. Another option is to use heuristics such as the minimum expansion tree in order to represent flows [12]. However, at the present time, these networks are applied less frequently than mesh networks. In real life, the principal problem that ramified networks present is ruptures in tubing, causing loss of service in several points in the network. This happens because there is only one route between one point and another. In a mesh network, the interruption of service due to ruptures in tubing happens less frequently. The design of these networks allows water to arrive to its destination via several trajectories. For this reason a break in tubing does not usually gravely affect other points in the network. In spite of fact that the implementation cost of mesh networks is higher than that of ramified networks, their use is justified by their increased reliability [14]. In order to find a solution for a mesh network, heuristic methods are used. For small instances, exact methods do not exist that solve the problem in polynomial time. This is because the number of possible solutions to analyze grows exponentially with the growth of the mesh. For this reason, heuristic methods promise to be a good alternative to finding solutions close to the optimum in polynomial time. In this work, a mapping is made of the Water Distribution Problem to the Job Shop Scheduling Problem, resulting in a problem of hydraulic networks with a mesh network.

The contribution of this work is the empirical verification that the Water Distribution problem is an NP-complete problem, given the hypothesis that the problem is bound by the group of constraints of the Job Shop Scheduling Problem. This problem is well known in literature by its acronym JSSP. JSSP is a problem that has been studied rigorously by the scientific community [16]. Mapping to a well-known problem allows reference to a wide information base in the literature of solution techniques. With a few changes to the Hydraulic Networks Problem, better results may be obtained in the study of this problem and in the search for improved efficacy and efficiency of algorithms used for real Hydraulic Networks Distribution problems.

The present document is organized in the following way:

Section 1 is the introduction. Section 2 describes the Hydraulic Networks Problem, complete with an explanation of the problem and the concepts used to exemplify a Hydraulic Network Distribution. Section 3 defines the Job Shop Scheduling Problem and the Hydraulic Network Problem in a Water Distribution System. It explains the analogies presented and the relationships that exist between both problems. An instance of the problem is shown in a disjunctive graph. Section 4 presents the empirical mapping of JSSP to Water Distribution Network Problem and the resulting mathematical model of Water Distribution Network. Section 5 describes future work that will provide continuity to this investigation. Section 6 asserts the conclusions of this work.

2. Hydraulic Networks Problem

The Hydraulic Networks Problem in a Water Distribution System is explained in this section.

A set of supply sources delivers water to cities. In order to achieve the distribution, a series of steps is followed. Each step involves using one source for a period of time in order to give service to a set of cities. The cities are made up of colonies and each colony contains the final users of the network, those to which the supply service is destined.

Considering a set of supply sources and a set of cities, one program is the assignment that fixes an interval of time in order to offer service to each user. The problem consists of finding a program that carries out an efficient distribution to all the users of the network. The service distribution should be carried out in the least possible time, while satisfying the requirements of the users. In order to minimize the service time it is necessary to ensure that the resources are actually arriving to the users. In this way, a good distribution is reflected and optimum coverage and saving service time are achieved.

3. Water Networks Model and *JSSP*

In the mathematical model, the objective function consists of minimizing the distribution cost of water. The model constraints are: the resource capacity, the physical design of the network, and the readiness of the resources. In order to propose a mapping, an analogy is made between the problems of hydraulic networks and JSSP.

In the Water Distribution Network Problem there is a set of supply sources $F = \{ F_1, F_2,..., F_m\}$ and a set of cities $C =\{C_1, C_2, C_3,..., C_n\}$. Each city is made up of a set of colonies. In the colonies, the final users are those to which the supply service of water is destined. In order to complete the service in a city, each supply source should provide service to the colonies respecting the physical precedence. The precedences are determined by the location of the colonies and by the design of the mesh network. In this problem, a supply source could give service to several colonies and the colonies could be assisted by several supply sources at several different times. The objective of water network problems is to minimize the distribution cost, while guaranteeing efficient distribution in which all the cities can be assisted. Figure 1 presents an example of a mesh hydraulic network. There are three supply sources *F1*, *F2*, and *F3*. Each source gives service to one city from the set *{C1, C2, C3}*, but has the possibility to give service to another city. Each city is formed by a set of colonies.

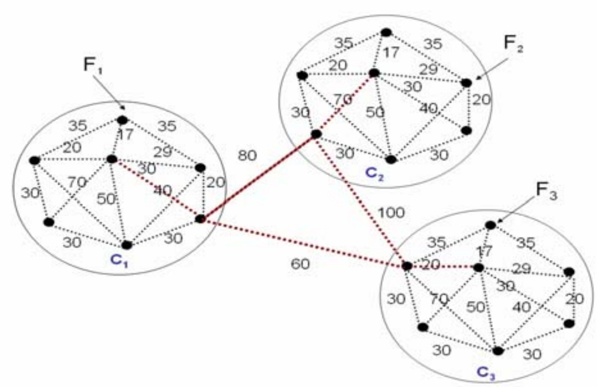

Figure 1. Mesh Hydraulic Network

The JSSP problem can be represented by means of a disjunctive graph $G = (V, A, E)$ [15], such as the one shown in Figure 2. This graph is made up of a set of arcs A, a set of edges E and a set of vertexes V. The vertexes 0 and f represent the initial and final operations of all jobs. The set V defines the set of operations O. Each operation is represented by a vertex $i \in V$. The set A is made up of subsets that each represent one job, J_j, which requires the execution of a subset of operations of V. For each pair of subsequent operations in the same job $(i, i') \in A$, there is an arch that shows direction, which implies a precedence constraint of an operations pair (i, i'), $i \prec i'$. The set E is made up of subsets, each of which represent a subset of operations of V where each subset is executed by one machine m_k. Resource capacity constraints exist in each machine. For each operations pair $\{i, i'\} \in E$, executed in the same machine m_k, an edge without direction exists that indicates that the pair $\{i, i'\}$ does not have a sequence of execution. Each node i has an associated weight p_i that indicates the time of execution of the operation i [3].

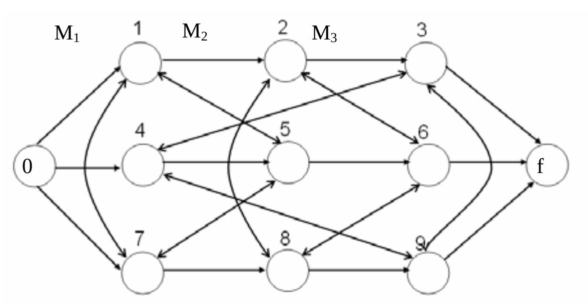

Figure 2. Disjunctive Graph of JSSP

One solution to JSSP consists of the selection of the order in which the operations should be carried out in each machine, which means selecting the direction of each edge of $\{i, i'\} \in E$. This generates precedence constraints in the execution of the operations pair in the same machine. The resulting digraph is acyclic. The total longitude of the longest path between 0 and f is the minimum. That is, the objective function minimizes the maximum completion time of the last job in the system.

As in JSSP, the water distribution problem in hydraulic networks can be represented by a disjunctive graph, as shown in Figure 3. The disjunctive graph $G = (V, A, E)$ consists of a set of vertexes V, a set of arcs A, and a set of edges E.

Nodes (vertexes) represent the sources of demand. Three cities exist C_1, C_2, C_3. Each city consists in turn of three colonies c: $C_1 = \{c_1, c_2, c_3\}$, $C_2 = \{c_4, c_5, c_6\}$ and $C_3 = \{c_7, c_8, c_9\}$.

Each colony implicitly contains the final users of the network.

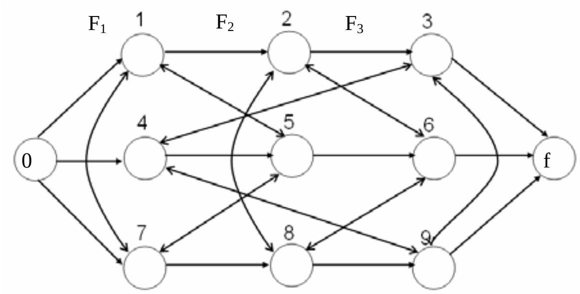

Figure 3. Disjunctive Graph of Water Network

In the graph, the arcs define precedence constraints in the colonies C. There are nine disjunctive arcs that represent the resource capacity constraints of the supply sources. Three supply sources exist, F_1, F_2, F_3 that supply the cities. Each source F_k can begin the supply in any colony of several cities.

A very important aspect that should be considered is what happens when a supply source F_1 cannot continue supplying the users. In this case, the service to users from the source F_1 could come from another supply source, for example F_2, as long as the resources of the source F_2 are sufficient to satisfy the minimum requirements of the users. For the mapping in question, this would not be possible. In order to make the model JSSP, it would have to relax. This relaxation would indicate that each source could give service to any colony in the system. This would be the equivalent to a special model of the JSSP called Total Flexible JSSP (TFJSSP) [16]. The literature indicates that the TFJSSP continues to be NP-complete, in spite of the more relaxed restrictions [17].

4. Mapping of the Hydraulic Network Problem

The Water Distribution problem in a Mesh Network can be viewed as an analogy to problem of JSSP (Job Shop Scheduling Problem).

JSSP is thoroughly studied in the combinatorial optimization area. Informally, it can be explained that JSSP consists of finding the optimum assignment of operations of jobs to available machines. The machines process the operations.

One objective in the solution of JSSP, is to achieve an optimum scheduling of jobs in machines in such a way that the processing time is minimized. The schedule must respect the processing time defined for each operation and the priority of execution of the operations. It should also consider the availability of machines.

By drawing this analogy, it can be seen that the optimum distribution problem in a water network could be solved by means of the same procedures used to solve scheduling problems in JSSP. Upon applying the procedures of JSSP to the Network Hydraulics Problem, a configuration and efficient distribution of resources for each of the users of the net can be obtained.

The following are some essential elements needed in order to understand the water distribution problem and the analogy drawn here:

1. The supply sources are equivalent to machines in JSSP, $m_k = F_k$. The sources are the entities that provide a service to the users. There is a group of them, and it is assumed that any supply source can only offer only one service at a time. That is, one source could only supply one part of the mesh network at any given time.
2. The Colonies are equivalent to *i* operations in JSSP. They are the parts of the mesh network to which the supply is destined. In a mesh network, the colonies are like subsets, where each subset forms one city C_j.
3. Precedence constraint in cities (equivalent to precedence of operations in JSSP) means that before the service arrives to a colony c_i of users, it can go through another colony of users if the physical location requires it. The order of attention to the colonies follows the supply trajectory to the users. The trajectory may vary since the flow can be bidirectional in mesh networks. In this case, the flow is defined unidirectional.
4. A set of colonies generally has a well-defined mesh network through which service is provided to the users. Each source F_k provides service to only one colony of each city. If it provides service to more than one colony in a city, this is equivalent to a Flexible JSSP.
5. Distribution is equivalent to scheduling in JSSP. It is a service assignment that is carried out within a given interval of time.
6. Operative Interval is equivalent to the objective function of makespan in JSSP. It is the period or lapse of time needed to complete the supply to all the cities. It achieves a greater coverage, giving service to all the users of the network. At the same time it minimizes the time required for this covering. This concept is also known as program.
7. Diameters of tubing. The ideal diameter is considered in such a way that it does not influence the time of distribution of the fluid.

Based on the generated mapping, the following is the mathematical model of disjunctive programming for the problem of hydraulic networks:

$$C = \{C_1, C_2, \ldots, C_n\}$$

$$F = \{F_1, F_2, \ldots, F_m\}$$

$$c = \{c_1, c_2, \ldots\}$$

$$C_k \subseteq c$$

$$F_k \subseteq c$$

$$\min\left(\max_{c_I i \in c}\left(s_{c_i} + p_{c_i}\right)\right) \qquad (1)$$

$$\forall\ c_i \in c \qquad s_{c_i} \geq 0 \qquad (2)$$

$$\forall\ c_i, c_j \in c,\ \forall (c_i \prec c_j) \in C_i \qquad s_{c_i} + p_{c_i} \leq s_{c_j} \qquad (3)$$

$$\forall\ c_i, c_j \in c,\ \forall\left(c_i, c_j \in F_k\right) \qquad s_{c_i} + p_{c_i} \leq s_{{}_j c_j} \vee s_{c_j} + p_{c_j} \leq s_{c_i} \qquad (4)$$

In the mathematical model, it is assumed that the pumps that pump the fluid from the sources do not have enough power to provide simultaneous service to all the colonies, so interruptions must be made in the service, so service is only provided to one colony at a time. The hydraulic networks model presents a set of cities C, a set of supply sources F, and a set of colonies c, which need to be supplied with water. Each city C_k is formed by a subset of colonies of the set c. Each supply source F_k should supply a subset of colonies of the set c. The objective function in (1) minimizes the period of time in order to deliver the complete supply to all the cities. The supply time for each colony is the time from the beginning of service in the colony c_i, plus the required service time p_{ci} in c_i. This model is subject to three sets of constraints. The constraints in (2) indicate that the start time of service in each colony is positive. The constraints in (3) indicate that within a city, a precedence constraint exists for the assignment of the start of service for each colony. This means that the service in colony ci must be finished before the service in colony cj begins. This implies that at any given moment, in a city Ck, only one colony will have service, while the other colonies wait for their turn according to their precedence. This can be justified because the source pumps do not have sufficient power to simultaneously pump to all the colonies, so interruptions in service are necessary. The constraints in (4), are resource capacity constraints that indicate that an optimum service assignment should be designated for the set of colonies c_i that receive the service from a source F_k. The same problem presents itself, that the source pumps are unable to pump simultaneously to all the colonies that receive the service of F_k.

5. Future Work

In order to give continuity to this work it is necessary to deepen the mapping of the problem to a model of Flexible JSSP. In this way, there is correspondence between all of the hard restrictions from the Water Distribution Problem in Hydraulic networks to the hard restrictions from the NP problem. The purpose of this is to make the fewest possible modifications to the algorithms when applying resource assignment techniques from the Job Shop Scheduling Problem to the Water Distribution Problem.

6. Conclusions

A mapping of the Job Shop Scheduling Problem to the Water Distribution Problem is presented. From the mapping done here, it is concluded that the Water Distribution Problem in Hydraulic Networks is of NP-complete type with the constraints defined in the proposed model.

The importance of this work rests in the possibility of applying techniques that have been thoroughly used and accepted in JSSP for their high performance to the Water Distribution problem in Hydraulic Networks. By using these thoroughly proven JSSP techniques, better results could be achieved for the Water Distribution problems.

With the use and adaptation of these techniques, one could expect to positively impact the solution of Water Distribution problems if the constraints of the model are similar to those of the model Total Flexible JSSP.

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